§5.4 Work

Work = Force times distance

$= kg\text{-}m^2/s^2$ OR $\frac{kg\text{-}m}{s^2} \cdot m$ OR Newton-meters OR foot-pounds

Joules

No surprise, force will be a function and we will be integrating, with dx our typical incremental distance.

A typical variable-force function would be a hanging cable you're lifting.

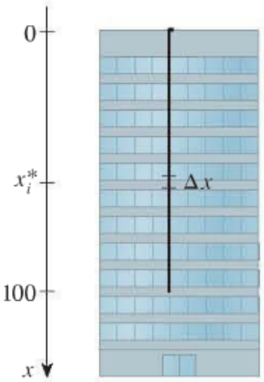


FIGURE 2

V **EXAMPLE 4** A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

we formulate this the book way & then do it my upside-down way

$$\text{Density} = \frac{200 \text{ lbs}}{100 \text{ ft}} = 2 \text{ lbs/ft}$$

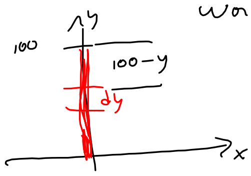
$$\text{Density} \cdot \text{length} = \text{FORCE} = (2)(dx)\,\frac{\text{lbs}}{\text{ft}}\cdot\text{ft} = 2\,dx \text{ lbs}$$

$$\text{work} = \text{Force} \times \text{Distance}$$
$$= (2dx)(x)$$
$$= 2x\,dx$$

$$\sum_{k=1}^{n} 2x_k\,dx \xrightarrow{n\to\infty} \int_0^{100} 2x\,dx$$

I always do things "right-side up"

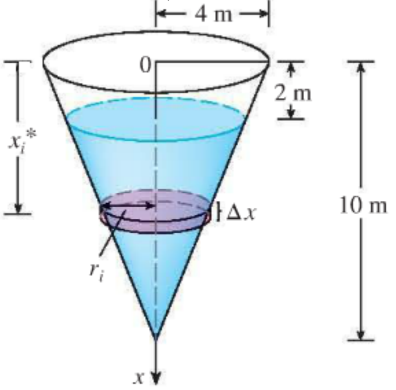
$$\text{work} = \int_0^{100} 2(100-y)\,dy$$

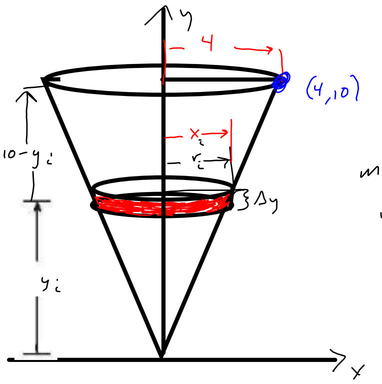


EXAMPLE 5 A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m³.)

mass = Density times volume

mass times gravity = force





$m = \frac{10}{4} = \frac{5}{2}$

$y = \frac{5}{2}x$

$x = \frac{2}{5}y$

$V_i = \pi r_i^2 \Delta x \quad (m^3)$

$m_i = \text{density} \cdot V_i \quad (kg = \frac{kg}{m^3} \cdot m^3)$

$= 1000 \cdot \pi r_i^2 \Delta x$

Force = mass times acceleration

$= (1000\pi r_i^2 \Delta x)(9.8m/s^2) = F$

Distance $= x_i = D$

work done on one layer

is $FD = 1000 \cdot 9.8\pi\, r_i^2 x_i \Delta x$

We just need to express r_i as a function of x:

$\int_0^{10}$

$W = \int_0^{10} (1000)(9.8)(\pi(\frac{2}{5}y)^2)(10-y)\,dy$

$= (9800\pi)(2/5)^2 \int_0^{10} y^2(10-y)\,dy$

$\pi r_i^2 \Delta y$

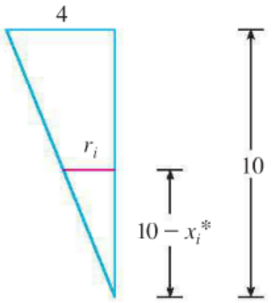
$m_i = 1000\pi\, r_i^2 \Delta y$

$F = 1000\pi\, r_i^2 \Delta y \cdot 9.8$

$= 9800\pi\, r_i^2 \Delta y$

$= 9800\pi\, (\frac{2}{5}y_i)^2 \Delta y$

$W = FD = (9800)(\frac{2}{5})^2 \pi\, y_i^2 (10-y_i)\Delta y$



$\frac{r_i}{10-x_i} = \frac{4}{10} = \frac{2}{5}$

$r_i = \frac{2}{5}(10-x_i) \Rightarrow r_i^2 = (\frac{2}{5})^2(10-x_i)^2$

$Work_i = (9.8)(1000)(\frac{2}{5})^2\pi\,(10-x_i)^2 x_i \Delta x$

$\Rightarrow Work = (9800)(\frac{2}{5})^2\pi \int_0^{10} (10-x)^2 x\,dx$

$= \frac{3920000}{3}\pi$

$9800 * \left(\frac{2}{5}\right)^2 * \text{Pi} \cdot \int_0^{10} (10-x)^2 \cdot x\,dx$

$9800 \cdot \left(\frac{2}{5}\right)^2 \cdot \text{Pi} \cdot \int_0^{10} y^2 \cdot (10-y)\,dy$

$1568\pi \left(\int_0^{10} (10-x)^2 x\,dx\right) = \frac{3920000}{3}\pi$

$1568\pi \left(\int_0^{10} y^2 (10-y)\,dy\right) = \frac{3920000}{3}\pi$