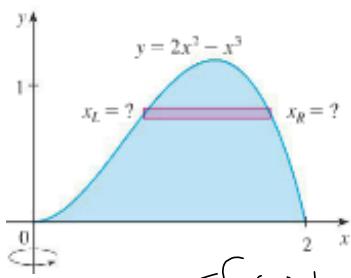


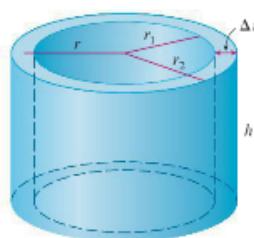
## Section 5.3 - Volumes by Cylindrical Shells

No quintic formula ( $5^{\text{th}}$  degree)

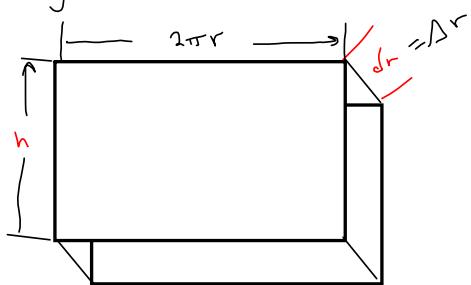


Finding volume of the solid obtained by revolving the area shown about the  $y$ -axis is not easy. There is a cubic formula (and a quartic formula), just like with the quadratic formula, so it is doable. It's just not easy.

There is an easier way for these...

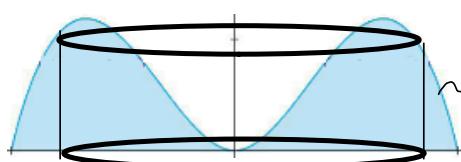


$\pi \int_0^1 ( ) dy$  is too tough.



$$\nearrow 2\pi rh \Delta r \rightarrow 2\pi \int_a^b r h dr$$

oriented as this one is



$$2\pi \int x f(x) dx$$

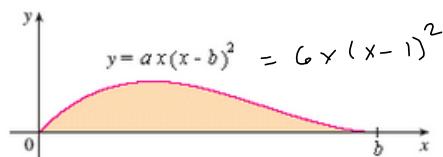
$$\begin{aligned} &\sim y = 2x^2 - x^3 \\ &2\pi \int_0^2 x (2x^2 - x^3) dx \\ &2x^2 - x^3 = 0 \\ &x^2(2-x) = 0 \end{aligned}$$

volume by shells!

## 1. Question Details

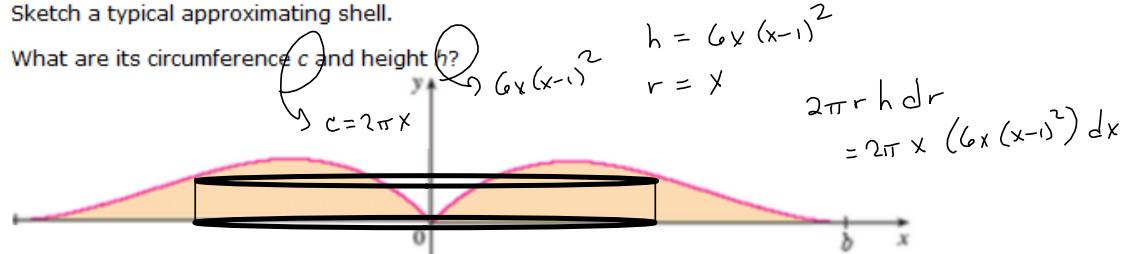
SCalc8 5.3.001. [335370]

Let  $S$  be the solid obtained by rotating the region shown in the figure about the  $y$ -axis. (Assume  $a = 6$  and  $b = 1$ .)



Sketch a typical approximating shell.

What are its circumference  $c$  and height  $h$ ?



$$h = 6x(x-1)^2$$

$$r = x$$

$$2\pi r h dr$$

$$= 2\pi x (6x(x-1)^2) dx$$

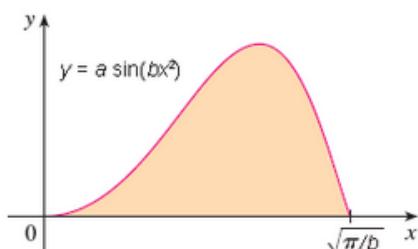
$$2\pi \int_0^1 6x^2(x^2 - 2x + 1) dx = 2\pi \int_0^1 (6x^4 - 12x^3 + 6x^2) dx$$

$$= 2\pi \left[ \frac{6}{5}x^5 - 3x^4 + 2x^3 \right]_0^1 = \left[ \frac{6}{5} - 3 + 2 \right](2\pi) = \frac{6-15+10}{5}\pi = \boxed{\frac{11}{5}\pi}$$

## 2. Question Details

SCalc8 5.3.002.

Let  $S$  be the solid obtained by rotating the region shown in the figure about the  $y$ -axis. (Assume  $a = 4$  and  $b = 2$ .)

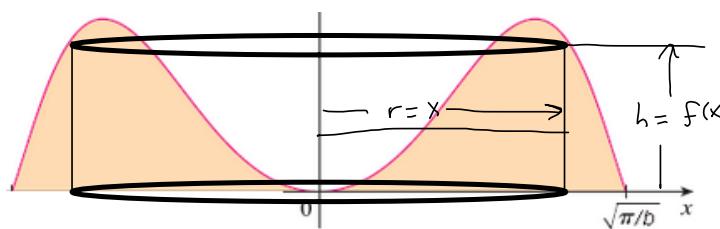


$$\begin{aligned} & 2\pi r h \Delta r \\ & 2\pi \times f(x) dx \\ & 2\pi \int_0^{\frac{\pi}{2}} x (4 \sin(2x^2)) dx \end{aligned}$$

Sketch a typical approximating shell.

Find its circumference  $c$  and height  $h$ .

Use Cylindrical Shells to find the volume  $V$  of  $S$ .



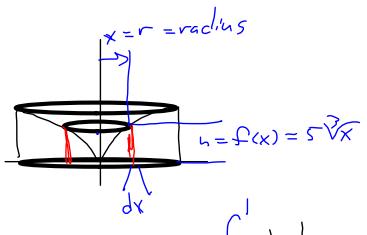
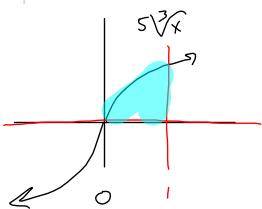
$$\begin{aligned} u &= 2x^2 \Rightarrow du = 4x dx \\ 2\pi & \int_0^{\frac{\pi}{2}} (\sin(2x^2)) (4x dx) \\ &= 2\pi \left[ -\cos(2x^2) \right]_0^{\frac{\pi}{2}} \\ &= 2\pi \left[ -\cos\left(2 \cdot \frac{\pi}{2}\right) - (-\cos(2 \cdot 0)) \right] \\ &= 2\pi [0 + 1] \left[ \frac{2\pi}{2} \right] \end{aligned}$$

## 3. Question Details

SCalc8 5.3.003. [3353669]

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the  $y$ -axis.

$$y = 5\sqrt[3]{x}, \quad y = 0, \quad x = 1$$



$$2\pi \int_0^1 r h dx$$

$$= 2\pi \int_0^1 x (5\sqrt[3]{x}) dx$$

$$= 10\pi \int_0^1 x \cdot x^{\frac{1}{3}} dx = 10\pi \int_0^1 x^{\frac{4}{3}} dx = 10\pi \left[ \frac{3}{7} x^{\frac{7}{3}} \right]_0^1 = \boxed{\frac{30\pi}{7}}$$

## 4. Question Details

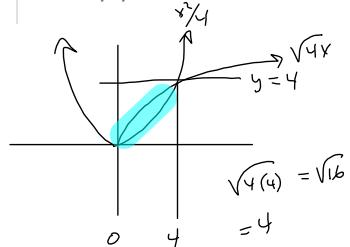
SCalc8 5.3.008. [3848583]

Let  $V$  be the volume of the solid obtained by rotating about the  $y$ -axis the region bounded by  $y = \sqrt{4x}$  and  $y = \frac{x^2}{4}$ . Find  $V$  by slicing.

Do it both ways!

Draw a diagram to explain your method.

Find  $V$  by cylindrical shells.



$$\frac{x^2}{4} = \sqrt{4x}$$

$$\frac{x^4}{16} = 4x$$

$$x^4 = 64x$$

$$x^4 - 64x = 0$$

$$x(x^3 - 64) \\ x=0 \quad x^3 - 64 = 0$$

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$$x^3 - 64 = 0$$

5. Question Details SCalc8 5.3.013. [3353639]

Use the method of cylindrical shells to find the volume  $V$  of the solid obtained by rotating the region bounded by the given curves about the  $x$ -axis.

$x = 2 + (y - 5)^2, x = 18$

Swap variables  
 $y = 2 + (x-5)^2, y = 18$ , about  $y$ -axis

Sketch the region and a typical shell. *if you want!*

$(x-5)^2 + 2 \leq 18$

$(x-5)^2 = 16$

$x-5 = \pm 4$

$x = 5 \pm 4$

$18 - (x-5)^2 - 2 = 16 - (x-5)^2$

$2\pi \int r h dx$

$= 2\pi \int_1^9 (16 - (x-5)^2) \times dx$

$= 2\pi \int_1^9 (16x - x^3 + 10x^2 - 25x) dx$

etc.

6. Question Details SCalc8 5.3.017. [33536]

Use the method of cylindrical shells to find the volume  $V$  generated by rotating the region bounded by the given curves about the specified axis.

$y = 5x - x^2, y = 4$ ; about  $x = 1$

## 7. Question Details

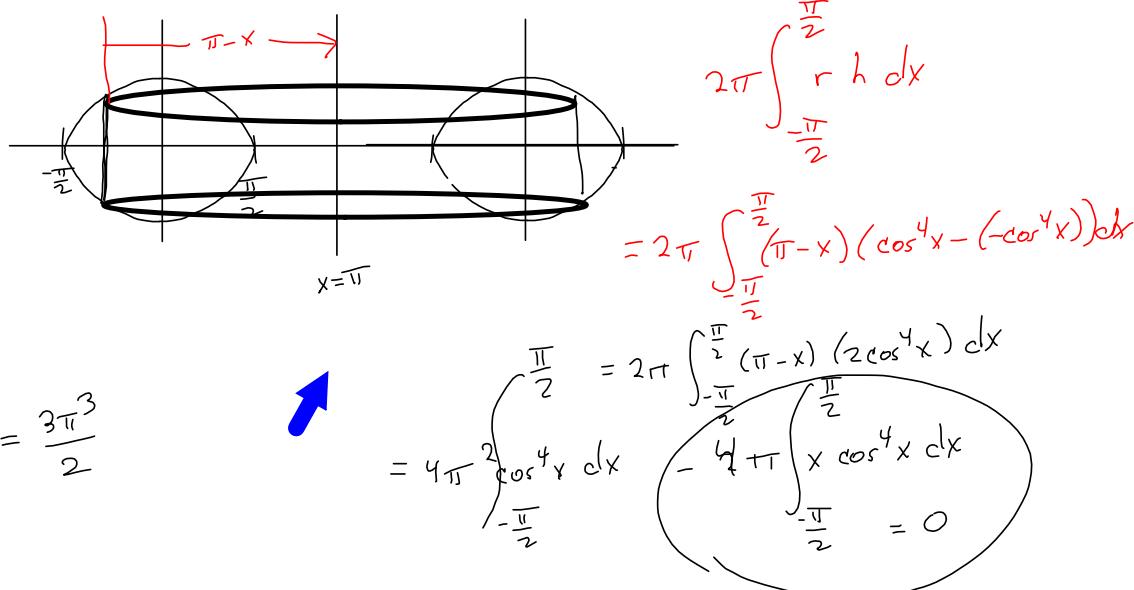
SCalc8 5.3.023. [3354]

Consider the following.

$$y = 6 \cos^4(x), \quad y = -6 \cos^4(x), \quad -\pi/2 \leq x \leq \pi/2; \quad \text{about } x = \pi$$

(a) Set up an integral for the volume  $V$  of the solid obtained by rotating the region bounded by the given curve about the specified axis.

(b) Use your calculator to evaluate the integral correct to five decimal places.



## 8. Question Details

SCalc8 5.3.031.

The integral represents the volume of a solid. Describe the solid.

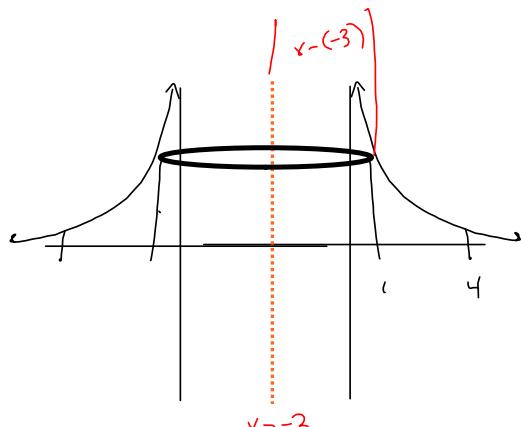
$$2\pi \int_1^4 \frac{4y + 3}{y^2} dy$$

## 8. Question Details

SCalc8 5.3.031.

The integral represents the volume of a solid. Describe the solid.

$$2\pi \int_1^4 \frac{y+3}{y^2} dy = 2\pi \int_a^b h dy = 2\pi \int_1^4 \frac{y(x+3)}{x^2} dx = 2\pi \int_1^4 (x-(-3)) \frac{1}{x^2} dx$$



$$2\pi \int_1^4 (x-(-3)) \left( \frac{1}{x^2} \right) dx$$

Revolution

$\frac{1}{x^2}$  from  $x=1$  to  $x=4$

about  $x = -3$

Revolution  $\frac{1}{y^2}$  from  $y=1$  to  $y=4$

about  $y = -3$

## 9. Question Details

SCalc8 5.3.041. [3353590]

The region bounded by the given curve is rotated about the specified axis. Find the volume  $V$  of the resulting solid by any method.

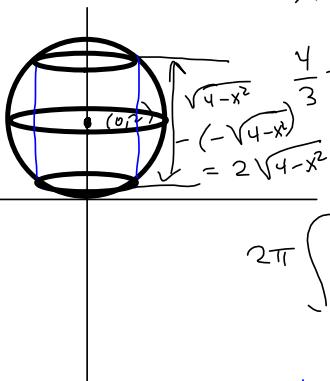
$$x^2 + (y - 2)^2 = 4; \text{ about the } y\text{-axis}$$

circle of radius  $r=2$

It's volume of sphere of  
radius  $r=2$ !

$$(y-2)^2 = 4-x^2$$

$$y-2 = \pm \sqrt{4-x^2}$$



$$\frac{4}{3}\pi (2)^3 = \frac{32\pi}{3} = \text{Volume!}$$

$$y = \sqrt{4-x^2} + 2$$

is top half  
Bottom half is

$$y = -\sqrt{4-x^2}$$

$$2\pi \int_0^2 r h = 2\pi \int_0^2 x (2\sqrt{4-x^2}) dx$$

$$u = 4-x^2$$

$$du = -2x dx$$

$$dx = \frac{du}{-2x}$$

$$= 4\pi \int_0^2 (\sqrt{4-x^2})(x dx)$$

$$= 4\pi \int_{x=0}^{x=2} (u)^{\frac{1}{2}} (x \cdot \frac{du}{-2x})$$

$$= -2\pi \int_{x=0}^{x=2} u^{\frac{1}{2}} du = -2\pi \left[ \frac{2}{3} u^{\frac{3}{2}} \right]_{0=x}^{2=x} = -2\pi \left[ \frac{2}{3} [4-x^2]^{\frac{3}{2}} \right]_0^2$$

$$-\frac{4}{3}\pi \left[ (4-4)^{\frac{3}{2}} - (4-0)^{\frac{3}{2}} \right] = -\frac{4}{3}\pi [8] = +\frac{32\pi}{3}$$

10.  Question Details

SCalc8 5.3.046.

Use cylindrical shells to find the volume  $V$  of the solid.

The solid torus (the donut-shaped solid shown in the figure) with radii  $r$  and  $R$

Meh.