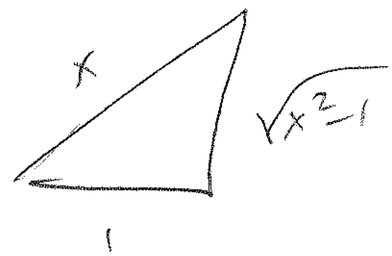
4# 202 TEST 3

4b $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}\,dx = I$

$x = \sec\theta \qquad dx = \sec\theta\tan\theta\,d\theta$

$x^2 = \sec^2\theta$

$$I = \int \frac{\sec^2\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta$$



$$= \int \sec^3\theta\,d\theta = I_2$$

$u = \sec\theta \qquad du = \sec\theta\tan\theta\,d\theta$

$dv = \sec^2\theta\,d\theta \qquad v = \tan\theta$

$$I_2 = uv - \int v\,du = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \ln|\sec\theta + \tan\theta|$$

$$\Rightarrow \int \sec^3\theta\,d\theta = \frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}{2}$$

$$= \frac{1}{2}\left[x\sqrt{x^2-1} + \ln\left|x + \sqrt{x^2-1}\right|\right] + C$$