2012 TEST 3

(1) $\int x^2 e^x \, dx$

10 pts

$= \boxed{x^2 e^x - 2xe^x + 2e^x + C}$

u		dv
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

(2) $\int_0^1 x\sqrt{1-x}\, dx = I$

10 pts

(a) Let $u = 1-x \Longrightarrow du = -dx \Longrightarrow dx = -du$

& $u - 1 = -x \Longrightarrow$

$1 - u = x$

$x = 1 \Longrightarrow u = 0$

$x = 0 \Longrightarrow u = 1$

This gives

$$I = \int_1^0 (1-u)\sqrt{u}\,(-du)$$

$$= \int_0^1 (1-u)u^{\frac{1}{2}}du = \int_0^1 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du$$

$$= \left[\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{5/2}\right]_0^1 = \frac{2}{3} - \frac{2}{5} = \frac{10-6}{15} = \boxed{\frac{4}{15}}$$

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(2) (b) 10 pts $I = \int_0^1 x\sqrt{1-x}\, dx = ?$

u	dv
x $+$	$(1-x)^{\frac{1}{2}}$
1 $-$	$-\frac{2}{3}(1-x)^{\frac{3}{2}}$
0	$\frac{2}{3}\cdot\frac{2}{5}(1-x)^{\frac{5}{2}}$

$$\Longrightarrow I = \left[-\frac{2}{3}x(1-x)^{\frac{3}{2}} - \frac{4}{15}(1-x)^{5/2}\right]_0^1$$

$$= -\frac{2}{3}(1)(0) - \frac{4}{15}(0) - \left[-\frac{2}{3}(0)(1) - \frac{4}{15}(1)\right] = \boxed{\frac{4}{15}}$$

(3) 10 pts $\int \cos^3 x\, dx = \int (1-\sin^2 x)\cos x\, dx$

$$= \int (\cos x - \sin^2 x \cos x)\, dx = \boxed{\sin x - \frac{\sin^3 x}{3} + C}$$

$\int_0^1 x\sqrt{1-x}\, dx$ $\quad u = x \quad du = dx \quad dv = (1-x)^{\frac{1}{2}} dx \quad v = -\frac{2}{3}(1-x)^{\frac{3}{2}}$

4a Karla's way

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx \qquad u = x^2 \quad du = 2x\,dx$$

$$x^2-1 = u-1$$

$$= \frac{1}{2}\int \frac{u\,du}{\sqrt{u-1}} \qquad v = u-1 \quad dv = du$$

$$u = v+1$$

$$= \frac{1}{2}\int \frac{(v+1)\,dv}{\sqrt{v}} = \frac{1}{2}\int \left(v^{\frac{1}{2}} + v^{-\frac{1}{2}}\right) dv$$

$$= \frac{1}{2}\left[\frac{2}{3}v^{\frac{3}{2}} + 2v^{\frac{1}{2}}\right] + C$$

$$= \frac{1}{3}v^{\frac{3}{2}} + v^{\frac{1}{2}} + C$$

$$= \frac{1}{3}(u-1)^{\frac{3}{2}} + (u-1)^{\frac{1}{2}} + C$$

$$= \frac{1}{3}(x^2-1)^{\frac{3}{2}} + (x^2-1)^{\frac{1}{2}} + C$$

4a My way

$u = x^2 - 1 \qquad du = 2x\,dx$

$u + 1 = x^2$

$$= \frac{1}{2}\int \frac{x^2 \cdot 2x\,dx}{\sqrt{x^2-1}} = \frac{1}{2}\int \frac{(u+1)\,du}{\sqrt{u}}$$

$$= \frac{1}{2}\int \left(u^{\frac{1}{2}} + u^{-\frac{1}{2}}\right) du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}\right] + C$$

$$= \frac{1}{3}u^{\frac{3}{2}} + u^{\frac{1}{2}} + C = \frac{1}{3}(x^2-1)^{\frac{3}{2}} + (x^2-1)^{\frac{1}{2}} + C$$

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4b

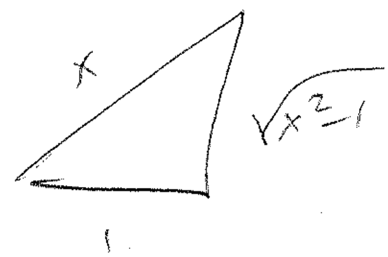
$$\int \frac{x^2}{\sqrt{x^2-1}}\,dx = I$$

$x = \sec\theta \qquad dx = \sec\theta\tan\theta\,d\theta$

$x^2 = \sec^2\theta$

$$I = \int \frac{\sec^2\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta$$

$$= \int \sec^3\theta\,d\theta = I_2$$



$u = \sec\theta \qquad du = \sec\theta\tan\theta\,d\theta$

$dv = \sec^2\theta\,d\theta \qquad v = \tan\theta$

$$I_2 = uv - \int v\,du = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \ln|\sec\theta + \tan\theta|$$

$$\Rightarrow \int \sec^3\theta\,d\theta = \frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}{2}$$

$$= \frac{1}{2}\left[x\sqrt{x^2-1} + \ln\left|x + \sqrt{x^2-1}\right|\right] + C$$

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx$$ 4b Josh's Prob Version

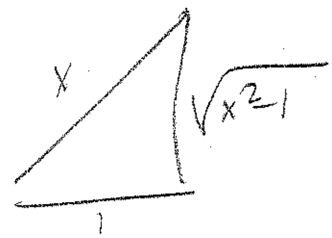
$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$$= \int \frac{\sec^3\theta\,\sec\theta\,\tan\theta\,d\theta}{|\tan\theta|}$$ Assume $\tan\theta > 0$

$$= \int \sec^4\theta\,d\theta = \int \sec^2\theta\,(\tan^2\theta + 1)\,d\theta$$

$$= \int (\tan^2\theta\,\sec^2\theta\,d\theta) + \int \sec^2\theta\,d\theta$$



$$= \frac{1}{3}\tan^3\theta + \tan\theta + C$$

$$= \frac{1}{3}\left(\sqrt{x^2-1}\right)^3 + \sqrt{x^2-1} + C$$

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5 10pts $\displaystyle\int \frac{dx}{x^2+3x+2} = \int \frac{dx}{(x+2)(x+1)} = I$

$$\frac{1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1} \implies$$

$$1 = A(x+1) + B(x+2)$$

$x=-1$: $x=-2$: $\longrightarrow$

$1 = B$ $1 = -A$

$$I = \int\left(-\frac{1}{x+2} + \frac{1}{x+1}\right)dx = -\ln(x+2) + \ln(x+1) + C$$

$$= \ln\left(\frac{x+1}{x+2}\right) + C$$

Probily should have $-\ln|x+2| + \ln|x+1| + C$,
but I won't be picky.

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(6) $\int_2^4 \frac{dx}{(x-1)^2}$ (a) $\Delta x = \frac{4-2}{4} = \frac{1}{2} \Rightarrow \frac{\Delta x}{2} = \frac{1}{4}$

(i) $x_0 = 2$ $x_2 = 3$ $x_4 = 4$

$x_1 = \frac{5}{2}$ $x_3 = \frac{7}{2}$

x	$\frac{1}{(x-1)^2}$
2	1
$\frac{5}{2}$	$\frac{4}{9}$
3	$\frac{1}{4}$
$\frac{7}{2}$	$\frac{4}{25}$
4	$\frac{1}{9}$

$\frac{1}{(\frac{3}{2})^2} = \frac{4}{9}$ $\frac{1}{(\frac{5}{2})^2} = \frac{4}{25}$

$\frac{1}{3^2} = \frac{1}{9}$ $\frac{1}{2^2} = \frac{1}{4}$

10 pts

Area $\approx \frac{1}{4}\left[1 + 2\left(\frac{4}{9}\right) + 2\left(\frac{1}{4}\right) + 2\left(\frac{4}{25}\right) + \frac{1}{9}\right]$

$= \frac{1}{4}\left[1 + \frac{8}{9} + \frac{1}{2} + \frac{8}{25} + \frac{1}{9}\right]$

$= \frac{1}{4}\left[2 + \frac{1}{2} + \frac{8}{25}\right] = \frac{1}{4}\left[\frac{100 + 25 + 16}{50}\right]$

$= \frac{1}{4}\left[\frac{141}{50}\right] = \boxed{\frac{141}{200} = .705}$

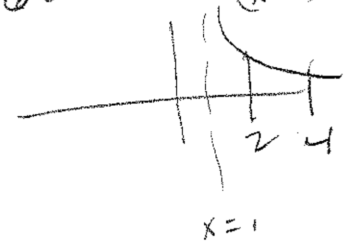
(ii) $|E_T| \le \frac{M(b-a)^3}{12n^2}$

$f(x) = (x-1)^{-2}$

$f'(x) = -2(x-1)^{-3}$

$f''(x) = 6(x-1)^{-4} = \frac{6}{(x-1)^4}$

$M \ge \max_{[2,4]} \{|f''(x)|\} = f''(2)$



$= \frac{6}{(2-1)^4} = \frac{6}{1^4} = \frac{6}{1} = 6 \Rightarrow$

$|E_T| \le \frac{(6)(2)^3}{12(4)^2} = \frac{6(8)}{(12)(16)} = \frac{1}{4} = .25$

$\boxed{|E_T| \le \frac{1}{4}}$

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(6) (a) (ii) Actual Error:

$$\int_2^4 (x-1)^{-2}\,dx = \left[-(x-1)^{-1}\right]_2^4 = \left[-\frac{1}{3} - \left(-\frac{1}{1}\right)\right]$$

$$= 1 - \frac{1}{3} = \frac{2}{3} = \text{Actual} = .\overline{6}$$

$$|\text{Actual Error}| = |.\overline{6} - .705| = \boxed{.038\overline{3}}$$

10 pts

(b) Simpson's $S_4 = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + \dots + 4y_3 + y_4)$

$$= \frac{\frac{1}{2}}{3}\left[1 + 4\left(\frac{4}{9}\right) + 2\left(\frac{1}{4}\right) + 4\left(\frac{4}{25}\right) + \frac{1}{9}\right]$$

$$= \frac{1}{6}\left[1 + \frac{16}{9} + \frac{1}{2} + \frac{16}{25} + \frac{1}{9}\right] = \frac{1}{6}\left[\frac{3}{2} + \frac{17}{9} + \frac{288}{25}\right]$$

$$= \frac{1}{6}\left[\frac{675 + 850 + 288}{(2)(9)(25)}\right] = \frac{1}{6}\left[\frac{1813}{450}\right] = \frac{1}{6}\left[\frac{1813}{450}\right]$$

$$= \frac{1813}{2700} = \boxed{.67\overline{148}}$$

(i) $|E_S| \le \frac{M(b-a)^5}{180n^4}$ where $M \ge \max_{[2,4]}\{|f^{(4)}(x)|\}$

$$|E_S| \le \frac{120(2)^5}{180(4)^4} = \frac{2}{3}\cdot\frac{2^5}{2^8} = \frac{1}{3\cdot 2^2} = \boxed{\frac{1}{12}}$$

$$\boxed{= .08\overline{3}} \ge |E_S|$$

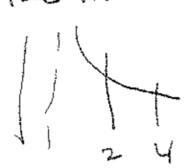
$f(x) = (x-1)^{-2}$

$f'(x) = -2(x-1)^{-3}$

$f^{(2)}(x) = 6(x-1)^{-4}$

$f^{(3)}(x) = -24(x-1)^{-5}$

$f^{(4)}(x) = 120(x-1)^{-6}$ max @ $x=2$: $\frac{120}{(2-1)^6} = 120$



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6b (ii) $|\text{Actual Error}| = \left| 1.\overline{6} - .67\overline{148} \right| = \frac{13}{2700}$

$$\boxed{= .00\overline{481} = |\text{Error}| = \frac{13}{2700}}$$

7a a $\displaystyle\int_2^\infty \frac{dx}{x(\ln x)^{2/3}}$ Diverges by previous homework & $\frac{2}{3} \le 1$, for a $p = \frac{2}{3} \le 1$ - test.

Hard Way:

$$\lim_{b\to\infty} \int_2^b (\ln x)^{-2/3} \cdot \frac{1}{x}\,dx = \lim_{b\to\infty} 3(\ln x)^{\frac{1}{3}} \Big]_2^b$$

$$= \lim_{b\to\infty} 3(\ln b)^{\frac{1}{3}} - 3(\ln 2)^{\frac{1}{3}} \quad \nexists$$ $(\ln x)^{\frac{1}{3}}$ grows w/o bound.

b $$\int_2^\infty \frac{dx}{x(\ln x)^{3/2}} = \lim_{b\to\infty} \int_2^b (\ln x)^{-3/2}\left(\frac{1}{x}\,dx\right)$$

$$= \lim_{b\to\infty} \left[-2(\ln x)^{-\frac{1}{2}}\right]_2^b = \lim_{b\to\infty} \left(-2(\ln b)^{-\frac{1}{2}}\right) - \left(-2(\ln 2)^{-\frac{1}{2}}\right)$$

$$= \boxed{\frac{2}{\sqrt{\ln 2}}} \approx \boxed{2.402244818}$$