202 E1

① $f(x) = x^2 - 4x - 7$

$= x^2 - 4x + 2^2 - 4 - 7$

$= (x-2)^2 - 11$

$(h,k) = (2, -11)$

$\mathcal{D} = [2, \infty)$

$\mathcal{R} = [-11, \infty)$

$(y-2)^2 - 11 = x$

$(y-2)^2 = x + 11$

$y - 2 = \pm\sqrt{x+11}$

$y = \boxed{\sqrt{x+11} + 2 = f^{-1}(x)}$

Take positive version.

$\boxed{\begin{array}{l} \mathcal{D} = [-11, \infty) \\ \mathcal{R} = [2, \infty) \end{array}}$

② ⓐ $(f^{-1})'(x) = \frac{1}{2}(x+11)^{-\frac{1}{2}}$

$(f^{-1})'(5) = \frac{1}{2}(16)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{4}\right) = \boxed{\frac{1}{8} = (f^{-1})'(5)}$

ⓑ $f'(x) = 2x - 4$

$f'(f^{-1}(5)) = f'(\sqrt{16} + 2) = f'(6)$

$= 2(6) - 4 = 12 - 4 = 8$ and so

$\boxed{(f^{-1})'(5) = \frac{1}{8}}$

202 E1

3 (a) $y = \ln(x^3 - 17x) - 5\ln(5x^2+27) \implies$

$$\boxed{y' = \frac{3x^2-17}{x^3-17x} - 5\left(\frac{10x}{5x^2+27}\right)}$$

(b) $\ln y = \ln x + \ln(x+1) + 3\ln(x-2)$

$- \ln(x^2+1) - \ln(2x+3) \longrightarrow$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{x+1} + \frac{3}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3} \longrightarrow$$

$$\boxed{y' = \left(\frac{1}{x} + \frac{1}{x+1} + \frac{3}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3}\right) \frac{x(x+1)(x-2)^3}{(x^2+1)(2x+3)}}$$

(c) $y = 5^{3x+2} \longrightarrow \boxed{y' = (\ln 5)(3)5^{3x+2}}$

(d) $\ln y = (x^2+2x)\ln(x^2+2x) \longrightarrow$

$$\frac{y'}{y} = (2x+2)\ln(x^2+2x) + \cancel{(x^2+2x)}\left(\frac{2x+2}{\cancel{x^2+2x}}\right) \implies$$

$$\boxed{y' = \left((2x+2)\ln(x^2+2x) + 2x+2\right)(x^2+2x)^{x^2+2x}}$$

202 E1

(3) (e) $y = \log_3 5 + \log_3 x + 2x \log_3 e \longrightarrow$

$y' = \boxed{\frac{1}{\ln 3} \cdot \frac{1}{x} + 2\log_3 e}$ or $\frac{1}{(\ln 3)x} + \frac{2}{\ln 3}$

(f) $y = \int_0^{x^2+3} \cos(5t)\,dt \longrightarrow$

$\boxed{y' = (2x)\cos(5(x^2+3))}$

$7^x = e^{\ln(7^x)} = e^{(\ln 7)x}$

(4) (a) $\int 7^x\,dx = \boxed{\frac{1}{\ln 7} 7^x + C}$

(b) $\int (x+1)\, 3^{x^2+2x}\,dx$

$= \frac{1}{2}\int 3^{x^2+2x}(2x+2)\,dx = \boxed{\frac{1}{2}\cdot\frac{1}{\ln 3}\cdot 3^{x^2+2x} + C}$

(5) $y'' = 2e^{-x}$

$y' = -2e^{-x} + C_1$

$y'(0) = -2 + C_1 = 3$

$\Rightarrow \boxed{C_1 = 5}$

$y = 2e^{-x} + 5x + C_2$

$y(0) = 4 = 2 + C_2 = 4$

$\longrightarrow C_2 = 2 \longrightarrow$

$y = \boxed{2e^{-x} + 5x + 2}$

202 E1

6 a $\lim_{x\to 1}\left(\frac{(x-3)(x-1)}{x-1} + \frac{x-1}{\cos(\frac{\pi}{2}x)}\right)$

$$= \lim_{x\to 1}(x-3) + \overset{L'H}{\lim_{x\to 1}} \frac{1}{-\frac{\pi}{2}\sin(\frac{\pi}{2}x)}$$

$$= 1-3 + -\frac{2}{\pi}\cdot\frac{1}{\sin\frac{\pi}{2}} = \boxed{-2-\frac{2}{\pi}}$$

b $\lim_{x\to\infty}\left(\left(1+\frac{4}{x}\right)^{\frac{4}{3x}}\right) = y \longrightarrow$

$$\ln y = \lim_{x\to\infty}\left(\frac{4}{3x}\ln\left(1+\frac{4}{x}\right)\right)$$

$$\overset{L'H}{=} \lim_{x\to\infty}\frac{\ln(1+\frac{4}{x})}{\frac{3x}{4}} \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{-\frac{4}{x^2}}{1+\frac{4}{x}}}{\frac{3}{4}}$$

$$= \lim_{x\to\infty}\frac{\frac{-\frac{4}{x^2}}{\frac{x+4}{x}}}{\frac{3}{4}} = \lim_{x\to\infty}\frac{4}{3}\cdot\frac{x}{x+4}\cdot\left(-\frac{4}{x^2}\right) = 0$$

$$\Longrightarrow \boxed{y = e^0 = 1}$$

202 E1

$\boxed{B2}$ $H_0 = 200$ $H'(t) = k(H(t) - H_s)$

$H_s = 68$ Let $y = H - H_s$. Then

$y' = ky \implies$

$y = Ce^{kt}$

$y(0) = C = 200 + 68 = 134 \longrightarrow$

$H - H_s = 134e^{kt}$ Also know that

$y(1) = H(1) - 68 = 134e^{k} = 170 - 68$

$\implies 134e^{k} = 102 \longrightarrow$

$e^{k} = \frac{102}{134} \longrightarrow$

$k = \ln\left(\frac{102}{134}\right) \approx -.2728669867$

So, $y(t) = H(t) - 68 \approx 134e^{-.2728669867}$

$\implies H(t) \approx 134e^{-.278669867} + 68$

$\therefore H(t) \overset{\text{SET}}{=} 120 = 134e^{kt} + 68 \longrightarrow$

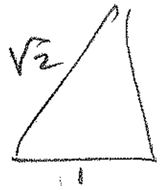
$134e^{kt} = 52 \longrightarrow$

$e^{kt} = \frac{52}{134}$ $\implies t = \frac{\ln(52/134)}{\ln(102/134)}$

$kt = \ln(52/134)$ $\approx \boxed{3.469075145 \text{ minutes}}$

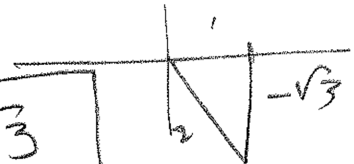
202 E1

(7) (a) $\sec^{-1}(\sqrt{2}) = \boxed{\frac{\pi}{4}}$

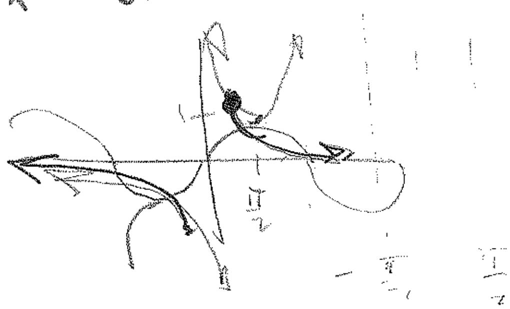


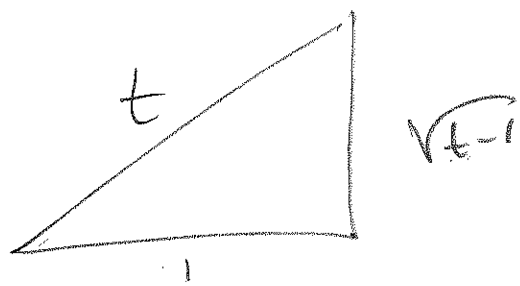
(b) $\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$

(c) $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \boxed{-\sqrt{3}}$



(d) $\lim_{x \to \infty} \csc^{-1}(x) = 0$





$\boxed{B1}$ $y = \tan^{-1}(\sqrt{t-1})$. We know $f(x) = \tan x$ has its $\tan^{-1} x$ as inverse. We know its derivative, $\sec^2 x$. Deriv. of inverse is

$$\frac{1}{\sec^2(\tan^{-1}(x))}$$

Apply Chain Rule

Convert:

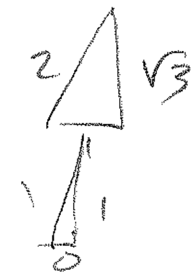
$$\frac{1}{\sec^2(\tan^{-1}(\sqrt{t-1}))} \cdot \frac{1}{2\sqrt{t-1}}$$

$$\frac{1}{t^2} \cdot \frac{1}{2\sqrt{t-1}} = \boxed{\frac{1}{2t^2\sqrt{t-1}}}$$

202 E1

B3

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(e^{\cot \omega} + 1\right) \csc^2 \omega \, d\omega = I$$



Let $u = \cot \omega$. Then $du = -\csc^2\omega \, d\omega$.

This gives $u\left(\frac{\pi}{2}\right) = 0$, $u\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

$$I = -\int_{\frac{1}{\sqrt{3}}}^{0} e^u \, du + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \csc^2\omega \, d\omega$$

$$= -\left[e^0 - e^{\frac{1}{\sqrt{3}}}\right] - \cot \omega \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 1 - e^{\frac{1}{\sqrt{3}}} - \left[0 - \frac{1}{\sqrt{3}}\right]$$

$$= \boxed{-1 + e^{\frac{1}{\sqrt{3}}} + \frac{1}{\sqrt{3}}}$$