

202 E 1

$$\begin{aligned} \textcircled{1} f(x) &= x^2 - 4x - 7 \\ &= x^2 - 4x + 2^2 - 4 - 7 \\ &= (x-2)^2 - 11 \end{aligned}$$

$$(h, k) = (2, -11)$$

$$D = [2, \infty)$$

$$R = [11, \infty)$$

$$(y-2)^2 - 11 = x$$

$$(y-2)^2 = x+11$$

$$y-2 = \pm \sqrt{x+11}$$

$$y = \sqrt{x+11} + 2 = f^{-1}(x)$$

Take positive version.

$$\begin{aligned} D &= [11, \infty) \\ R &= [2, \infty) \end{aligned}$$

$$\textcircled{2} \textcircled{a} (f^{-1})'(x) = \frac{1}{2}(x+11)^{-\frac{1}{2}}$$

$$(f^{-1})'(5) = \frac{1}{2}(16)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8} = (f^{-1})'(5)$$

$$\textcircled{b} f'(x) = 2x - 4$$

$$f'(f^{-1}(5)) = f'(\sqrt{16} + 2) = f'(6)$$

$$= 2(6) - 4 = 12 - 4 = 8 \text{ and so}$$

$$(f^{-1})'(5) = \frac{1}{8}$$

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$$\textcircled{3} \textcircled{a} y = \ln(x^3 - 17x) - 5 \ln(5x^2 + 27) \rightarrow$$

$$y' = \frac{3x^2 - 17}{x^3 - 17x} - 5 \left(\frac{10x}{5x^2 + 27} \right)$$

$$\textcircled{b} \ln y = \ln x + \ln(x+1) + 3 \ln(x-2) \\ - \ln(x^2+1) - \ln(2x+3) \rightarrow$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{x+1} + \frac{3}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3} \rightarrow$$

$$y' = \left(\frac{1}{x} + \frac{1}{x+1} + \frac{3}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3} \right) \frac{x(x+1)(x-2)^3}{(x^2+1)(2x+3)}$$

$$\textcircled{c} y = 5^{3x+2} \rightarrow y' = (\ln 5)(3) 5^{3x+2}$$

$$\textcircled{d} \ln y = (x^2 + 2x) \ln(x^2 + 2x) \rightarrow$$

$$\frac{y'}{y} = (2x+2) \ln(x^2+2x) + (x^2+2x) \left(\frac{2x+2}{x^2+2x} \right) \rightarrow$$

$$y' = ((2x+2) \ln(x^2+2x) + 2x+2) (x^2+2x)^{x^2+2x}$$

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③ e) $y = \log_3 5 + \log_3 x + 2x \log_3 e^x \rightarrow$

$$y' = \left[\frac{1}{\ln 3} \cdot \frac{1}{x} + 2 \log_3 e \right] \text{ or } \frac{1}{(\ln 3)x} + \frac{2}{\ln 3}$$

④ f) $y = \int_0^{x^2+3} \cos(st) dt \rightarrow$

$$y' = (2x) \cos(5(x^2+3))$$

$$7^x = e^{\ln(7^x)} = e^{(\ln 7)x}$$

④ g) $\int 7^x dx = \left[\frac{1}{\ln 7} 7^x + C \right]$

⑤ a) $\int (x+1) 3^{x^2+2x} dx$

$$= \frac{1}{2} \int 3^{x^2+2x} (2x+2) dx = \left[\frac{1}{2} \cdot \frac{1}{\ln 3} \cdot 3^{x^2+2x} + C \right]$$

⑤ $y'' = 2e^{-x}$

$$y' = -2e^{-x} + C_1$$

$$y'(0) = -2 + C_1 = 3 \rightarrow C_1 = 5 \rightarrow$$

$$\Rightarrow C_1 = 5$$

$$y = -2e^{-x} + 5x + C_2$$

$$y(0) = 4 = 2 + C_2 = 4$$

$$y = \left[2e^{-x} + 5x + 2 \right]$$

$$\textcircled{6} \lim_{x \rightarrow 1} \left(\frac{(x-3)(x-1)}{x-1} + \frac{x-1}{\cos\left(\frac{\pi}{2}x\right)} \right)$$

$$= \lim_{x \rightarrow 1} (x-3) + \lim_{x \rightarrow 1} \frac{1}{-\frac{\pi}{2} \sin\left(\frac{\pi}{2}x\right)} \quad \text{L'H}$$

$$= 1-3 + \frac{-\frac{3}{\pi}}{\sin \frac{\pi}{2}} = \boxed{-2 - \frac{2}{\pi}}$$

$$\textcircled{b} \lim_{x \rightarrow \infty} \left(\left(1 + \frac{4}{x}\right)^{\frac{4}{3x}} \right) = y \rightarrow$$

$$\ln y = \lim_{x \rightarrow \infty} \left(\frac{4}{3x} \ln \left(1 + \frac{4}{x}\right) \right)$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{4}{x}\right)}{\frac{3x}{4}} = \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{-\frac{4}{x^2}}{1 + \frac{4}{x}}}{\frac{3}{4}}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{-\frac{4}{x^2}}{\frac{x+4}{x}}}{\frac{3}{4}} = \lim_{x \rightarrow \infty} \frac{4}{3} \cdot \frac{x}{x+4} \cdot \left(-\frac{4}{x^2}\right) = 0$$

$$\rightarrow \boxed{y = e^0 = 1}$$

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B2

$$H_0 = 200 \quad H'(t) = k(H(t) - H_s)$$

$$H_s = 68 \quad \text{Let } y = H - H_s. \text{ Then}$$

$$y' = ky \implies$$

$$y = C e^{kt}$$

$$y(0) = C = 200 - 68 = 134 \implies$$

$$H - H_s = 134 e^{kt} \quad \text{Also know that}$$

$$y(1) = H(1) - 68 = 134 e^k = 170 - 68$$

$$\implies 134 e^k = 102 \implies$$

$$e^k = \frac{102}{134} \implies$$

$$k = \ln\left(\frac{102}{134}\right) \approx -0.2728669867$$

$$\text{So, } y(t) = H(t) - 68 \approx 134 e^{-0.2728669867 t}$$

$$\implies H(t) \approx 134 e^{-0.2728669867 t} + 68$$

$$\begin{matrix} \circ \\ \circ \\ \circ \end{matrix} \quad \text{SET} \quad H(t) = 120 = 134 e^{kt} + 68 \implies$$

$$134 e^{kt} = 52 \implies$$

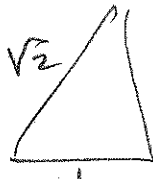
$$e^{kt} = \frac{52}{134}$$

$$\implies t = \frac{\ln(52/134)}{\ln(102/134)}$$

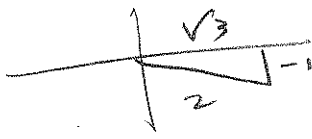
$$kt = \ln(52/134)$$

$$\approx \boxed{3.469075145 \text{ minutes}}$$

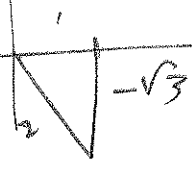
(7) (a) $\sec^{-1}(\sqrt{2}) = \frac{\pi}{4}$



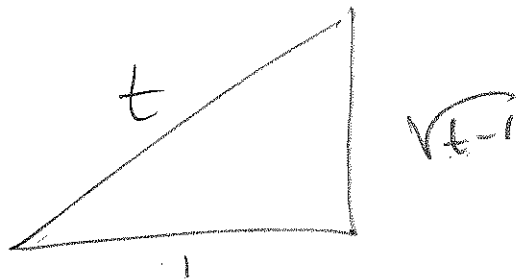
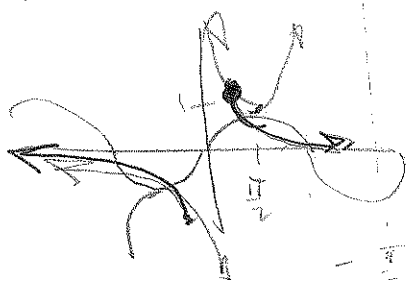
(b) $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$



(c) $\tan(\sin^{-1}(-\frac{\sqrt{3}}{2})) = -\sqrt{3}$



(d) $\lim_{x \rightarrow -\infty} \csc^{-1}(x) = 0$



B1 $y = \tan^{-1}(\sqrt{t-1})$. We know $f(x) = \tan x$

has this $\tan^{-1} x$ as its inverse. We know its derivative, $\sec^2 x$. Deriv. of its inverse is

$$\frac{1}{\sec^2(\tan^{-1}(x))}$$

Apply Chain Rule

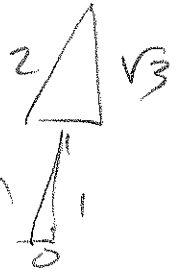
convert "

$$\frac{1}{\sec^2(\tan^{-1}(\sqrt{t-1}))} \cdot \frac{1}{2\sqrt{t-1}}$$

$$\frac{1}{t^2} \cdot \frac{1}{2\sqrt{t-1}} =$$


$$\boxed{\frac{1}{2t^2\sqrt{t-1}}}$$

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$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (e^{\cot \omega} + 1) \csc^2 \omega \, d\omega = I$$


Let $u = \cot \omega$ Then $du = -\csc^2 \omega \, d\omega$.

This gives $u(\frac{\pi}{2}) = 0$, $u(\frac{\pi}{3}) = \frac{1}{\sqrt{3}}$

$$I = - \int_{\frac{1}{\sqrt{3}}}^0 e^u \, du + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \csc^2 \omega \, d\omega$$


$$= - \left[e^0 - e^{\frac{1}{\sqrt{3}}} \right] - \cot \omega \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 1 - e^{\frac{1}{\sqrt{3}}} - \left[0 - \frac{1}{\sqrt{3}} \right]$$

$$= -1 + e^{\frac{1}{\sqrt{3}}} + \frac{1}{\sqrt{3}}$$