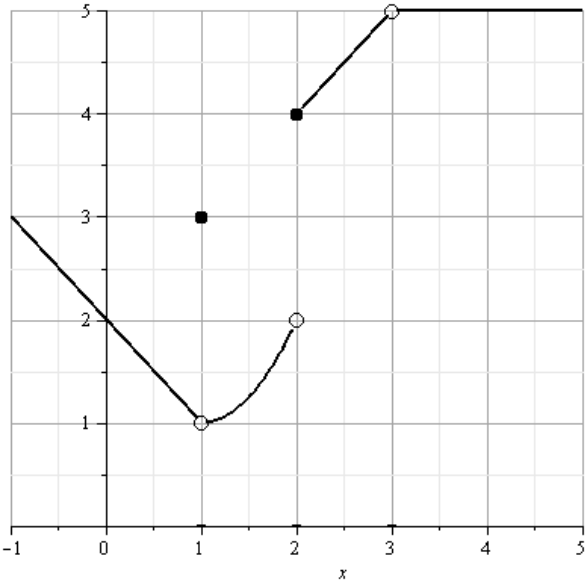
MAT 201 Final Test
Fall, 2012 Name_____

Do your work on separate paper. I will *not* grade this cover sheet. In fact, you should take the cover sheet *with* you at the end of the test.

1. The graph of a function f is given on the right. Employ the language (shorthand) of limits in your explanation(s), as needed.



   a. (5 pts) $\lim_{x\to 1} f(x)$

   b. (5 pts) $f(1)$

   c. (5 pts) Explain why $f(x)$ is not continuous at $x = 1$

   d. (5 pts) Explain why $\lim_{x\to 2} f(x)$ does not exist.

   e. (5 pts) Define $f(3)$ in such a way that f will be continuous at $x = 3$

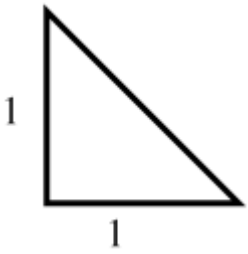
2. (10 pts) Use the precise definition of a limit to prove that $\lim_{x\to 4}(5x+4) = 24$.

3. (10 pts) Let $f(x) = x^2 - 5x + 2$. Find $f'(x)$ using the definition of the derivative.

4. (15 pts) Sketch the graph of $f(x) = x^3 - 5x^3 + 8x - 4$, showing all intercepts, max/min and inflection points.

5. (10 pts) Find the area between $f(x) = x^3 - 5x^3 + 8x - 4$ and $g(x) = x^2 - 3x + 2$.

6. (10 pts) A plane is flying one mile above the ground with an airspeed of 300 miles per hour. How fast is the distance from the observer to the plane changing when the plane is 1 mile East of the observer?



7. (10 pts) Use the linearization (or differentials) to approximate $\sqrt{39}$.

8. Find the area under the curve $y = 2x + 3$ on the interval [0,4] in two ways:

   a. (10 pts) By taking the limit of a Riemann sum. Again, use right endpoints.

   b. (10 pts) By the Second Fundamental Theorem of Calculus.

9. (10 pts) Find $\frac{d}{dx}\int_0^{\cos x}\sqrt{t+\sqrt{t}}\ dt$.

10. Evaluate each of the following definite integrals (5 pts each). Sketch the relevant region for each and discuss the difference between the two, in terms of areas (5 pts).

a. (10 pts) $\int_0^{\frac{5\pi}{3}} \sin x\,dx$

b. (10 pts **Bonus**) $\int_0^{\frac{5\pi}{3}} \left|\sin x\right| dx$

c. (10 pts) $\int x^3\left(2+x^4\right)^5 dx$

d. (10 pts) $\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\ dt$

11. (10 pts) Write the integral for the arc length of the function $f(x)=3x^2+2x$ over the interval from $x = 1$ to $x = 5$. Do not evaluate.

12. Write the integral for the volume of the solid of revolution obtained by rotating the region bounded by $x =1$, $x = 5$, and $y = 3x^2 + 2x$ about the...

a. (10 pts) ... x-axis.
b. (10 pts) ... y-axis

13. (10 pts) Write the integral for the surface area of the solid of revolution obtained by rotating the region bounded by $x =1$, $x = 5$, and $y = 3x^2 + 2x$ about the x-axis.

14. (10 pts) Find the force on one side (the *front* side) of a semicircular plate of radius 5 feet, as shown in the picture. Use 62.4 $\frac{lbs}{ft^3}$ for the weight density of water.