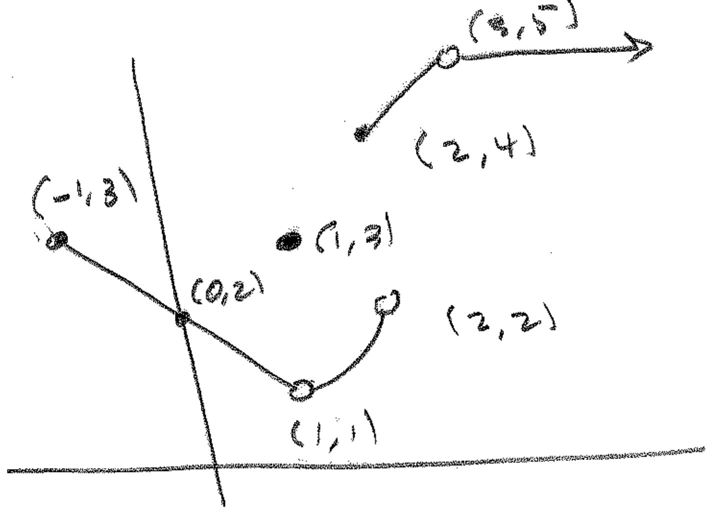
201 FINAL FALL '12

1



a 5pts $\lim_{x\to 1} f(x) = 1.$

b 5pts $f(1) = 3$

c 5pts $\lim_{x\to 1} f(x) = 1 \neq 3 = f(1)$ is why

f isn't cnt@ $x = 1.$

d 5pts $\lim_{x\to 2^-} f(x) = 2 \neq 4 = \lim_{x\to 2^+} f(x)$ is why

$\lim_{x\to 2} f(x) \nexists.$

5pts e Let $f(3) = 5$ to make f cnt @ $x = 3.$

2 10pts $\lim_{x\to 4} (5x+4) = 24.$

$\boxed{\text{Pf}}$ Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$. Then

$0 < |x-4| < \delta$ implies $|5x+4-24| = |5x-20|$

$= 5|x-4| < 5\delta = 5 \cdot \frac{\varepsilon}{5} = \varepsilon$ ■

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(3) 10 pts $f(x) = x^2 - 5x + 2 \Longrightarrow$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 2 - (x^2 - 5x + 2)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h + 2 - x^2 + 5x - 2}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = \frac{h(2x + h - 5)}{h} = 2x + h - 5 \xrightarrow{h \to 0} \boxed{2x - 5}$$

$$= f'(x)$$

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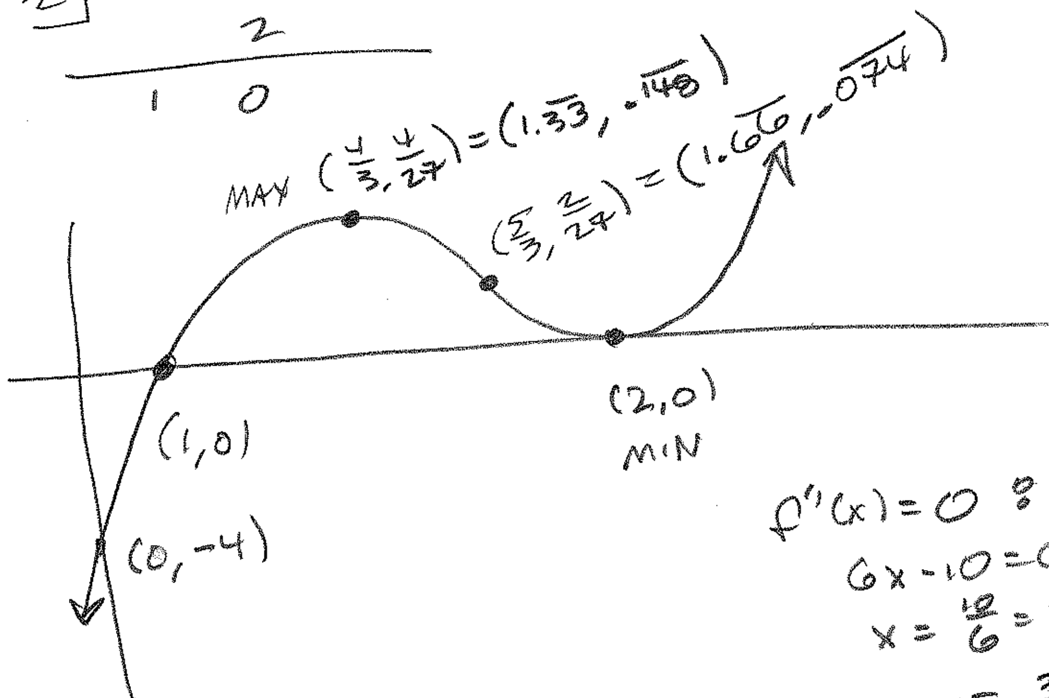
4. 15 pts $f(x) = x^3 - 5x^2 + 8x - 4$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 8 & -4 \\ & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & 0 \end{array} \quad x=1$$

$$\begin{array}{r|rrr} 2 & 1 & -4 & 4 \\ & & 2 & -4 \\ \hline & 1 & -2 & 0 \end{array} \quad x=2$$

$$\begin{array}{r|rr} 2 & 1 & -2 \\ & & 2 \\ \hline & 1 & 0 \end{array} \quad x=2$$

$f(x) = (x-1)(x-2)^2$



$f''(x) = 0$:
$6x - 10 = 0$ ⟹
$x = \frac{10}{6} = \frac{5}{3}$

$f'(x) = 3x^2 - 10x + 8$
$= (3x-4)(x-2) \stackrel{\text{SET}}{=} 0$
$x = \frac{4}{3}, 2$ are cps.

$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 5\left(\frac{4}{3}\right)^2 + 8\left(\frac{4}{3}\right) - 4$
$= \frac{64}{27} - 5\left(\frac{16}{9}\right) + \frac{32}{3} - 4$
$= \frac{64 - 240 + 288 - 108}{27} = \frac{4}{27} \doteq .\overline{148}$

$f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^3 - 5\left(\frac{5}{3}\right)^2 + 8\left(\frac{5}{3}\right) - 4 =$
$= \frac{125 - 375 + 360 - 108}{27}$
$= \frac{2}{27} \doteq .\overline{074}$

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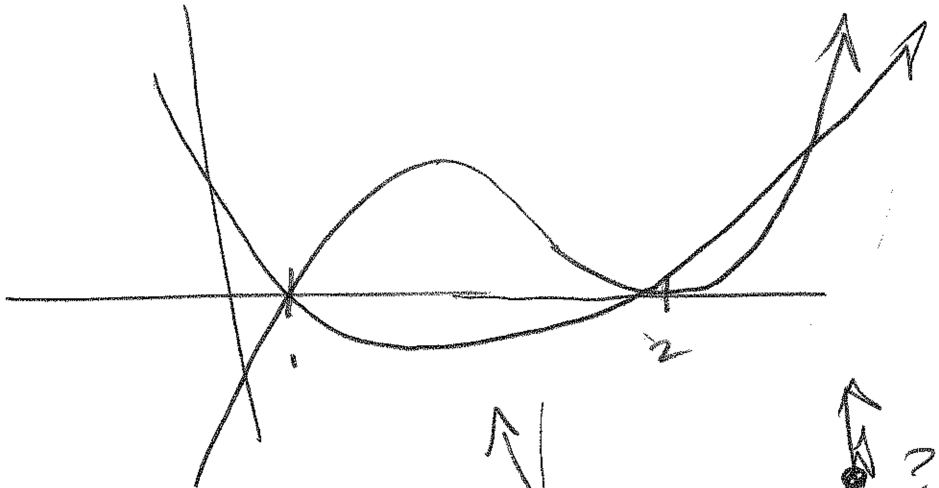
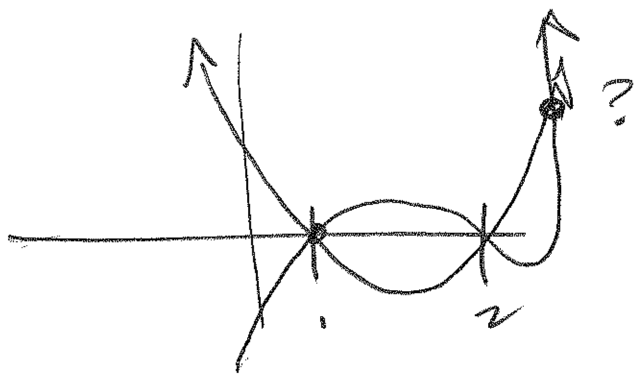
5 (10 pts) $f(x) = x^3 - 5x^2 + 8x - 4$

$g(x) = x^2 - 3x + 2 = (x-1)(x-2)$





Find intersection:

$x^3 - 5x^2 + 8x - 4 = x^2 - 3x + 2$ $\Rightarrow$

$x^3 - 6x^2 + 11x - 6 = 0$

1	1	-6	11	-6
		1	-5	6
2	1	-5	6	0
		2	-6	
	1	-3	0	

$x - 3 = 0$

$x = 3$ is other crosser.

$$\int_1^2 (x^3 - 6x^2 + 11x - 6)\,dx - \int_2^3 (x^3 - 6x^2 + 11x - 6)\,dx$$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_1^2$$

$$- \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_2^3$$

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5 cntd

$$= \left[\frac{2^4}{4} - 2\cdot(2)^3 + \frac{11}{2}(2)^2 - 6(2) - \left(\frac{1}{4} - 2 + \frac{11}{2} - 12\right)\right]$$

$$- \left[\frac{3^4}{4} - 2(3)^3 + \frac{11}{2}(3)^2 - 6(3) - \left(\frac{2^4}{4} - 2(2)^3 + \frac{11}{2}(2)^2 - 6(2)\right)\right]$$

$$= \left[\frac{16}{4} - 16 + \frac{44}{2} - 12 - \left(\frac{1}{4} + \frac{11}{2} - 14\right)\right]$$

$$- \left[\frac{81}{4} - 54 + \frac{99}{2} - 18 - \left(\frac{16}{4} - 16 + \frac{44}{2} - 12\right)\right]$$

$$= \frac{16+88}{4} - 28 - \left(\frac{1+22-56}{4}\right)$$

$$- \left[\frac{81+198}{4} - 34 - \left(\frac{16+88}{4} - 28\right)\right]$$

$$= \frac{104}{4} - 28 - \left(-\frac{33}{4}\right) - \left[\frac{279}{4} - 34 - \left(\frac{104}{4} - 28\right)\right]$$

$$= \frac{104}{4} + \frac{33}{4} - 28 - \frac{279}{4} + 34 + \frac{104}{4} - 28$$

$$= -\frac{38}{4} -$$

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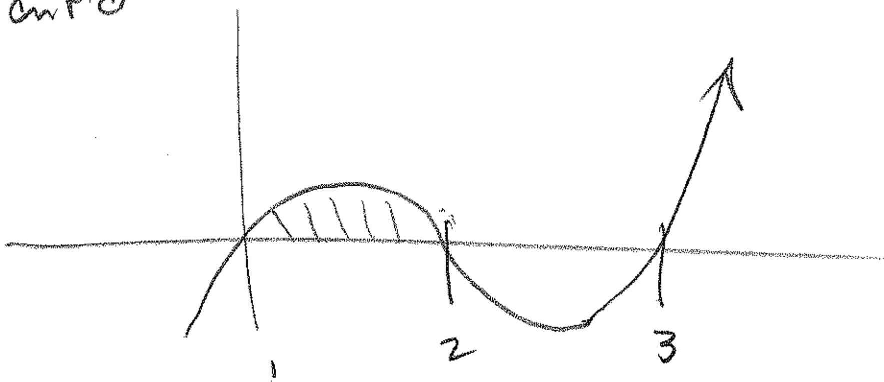
5 cont'd



$$\int_1^2 (x^3-6x^2+11x-6)\,dx - \int_2^3 (x^3-6x^2+11x-6)\,dx$$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_1^2 - \left[\left(\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right)\right]_2^3$$

$$= \frac{16}{4} - 16 + \frac{11}{2}(4) - 6(2) - \left(\frac{1}{4} - 2 + \frac{11}{2} - 6\right)$$

$$- \left[\frac{1}{4}(81) - 2(27) + \frac{99}{2} - 18 - \left(\frac{16}{4} - 16 + \frac{11}{2}(4) - 6(2)\right)\right]$$

$$= 4 - 16 + 22 - 12 - \left(\frac{1-8+22-24}{4}\right)$$

$$- \left[\frac{81}{4} - 54 + \frac{198}{4} - 18 - (4 - 16 + 22 - 12)\right]$$

$$= 26 - 28 - \left(-\frac{9}{4}\right) - \left[\frac{279}{4} - 72 - (26-28)\right]$$

$$= -2 + \frac{9}{4} - \frac{279}{4} + 70 = 68 - \frac{270}{4} = 68 - \frac{135}{2}$$

$$= \frac{136-135}{2} = \boxed{\frac{1}{2}}$$

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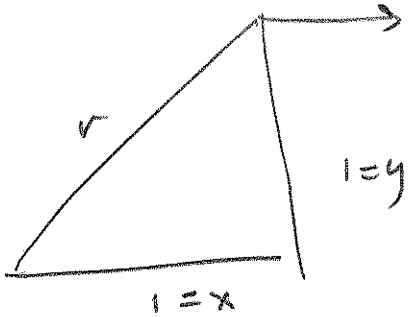

$\frac{dx}{dt} = 300$

$1 = y \quad \frac{dr}{dt} = ?$

we know $x^2 + y^2 = r^2$

10 pts

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

$$2(1)(300) + 2(1)(0) = 2\sqrt{2}\,\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{600}{2\sqrt{2}} = \frac{300}{\sqrt{2}} = \frac{300\sqrt{2}}{2} = 150\sqrt{2} \approx 212.132034 \text{ miles/hr}$$

7 $f(x) = \sqrt{x}$, $a = 36$, $\Delta x = 39 - 36 = 3$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}},$$

10 pts

$$f(a) = \sqrt{36} = 6$$

$$f'(a) = \frac{1}{2\sqrt{36}} = \frac{1}{2(6)} = \frac{1}{12}$$

$$f(a + \Delta x) \approx f(a) + f'(a)\Delta x$$

$$= 6 + \left(\frac{1}{12}\right)(3) = 6 + \frac{3}{12} = 6 + \frac{1}{4} = \boxed{6.25}$$

$$L(x) = f(a) + f'(a)(x - a)$$

$$= 6 + \frac{1}{12}(39 - 36)$$

$$= 6 + \frac{3}{12} = 6.25$$

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8 a 10 pts

$a = 0, b = 4, \quad \frac{b-a}{n} = \frac{4}{n} = \Delta x$

$x_k = a + k\Delta x = 0 + k \cdot \frac{4}{n} = \frac{4k}{n} \Longrightarrow$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} (2x_k + 3) \cdot \frac{4}{n} =$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(2\left(\frac{4k}{n}\right)+3\right) = \frac{4}{n}\left[\frac{8}{n}\sum_{k=1}^{n} k + 3\sum_{k=1}^{n} 1\right]$$

$$= \frac{4}{n}\cdot\frac{8}{n}\left(\frac{n^2 + \text{smaller}}{2}\right) + \frac{4}{n}\cdot 3 \cdot n$$

$$= \frac{32}{n^2}\left(\frac{n^2 + m}{2}\right) + 12$$

$$= 16\left(\frac{n^2+m}{n^2}\right) + 12 \xrightarrow{n \to \infty} 16 + 12 = \boxed{28}$$

~~9~~ 8b

$$\int_0^4 (2x+3)\,dx = \left[x^2 + 3x\right]_0^4 = 4^2 + 3(4) - (0^2 + 3(0))$$

$$= 16 + 12 = \boxed{28}$$

10 pts

9

$$\frac{d}{dx}\int_0^{\cos x} \sqrt{1+\sqrt{t}}\,dt$$

$$= \left(\sqrt{1+\sqrt{\cos x}}\right)(-\sin x)$$

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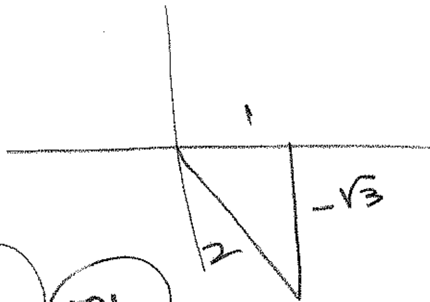
(10) (a) (10pts)

$$\int_0^{\frac{5\pi}{3}} \sin x \, dx = -\cos x \Big]_0^{\frac{5\pi}{3}}$$

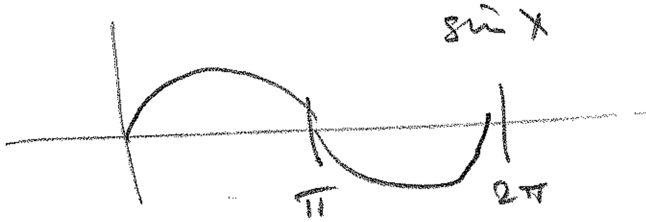


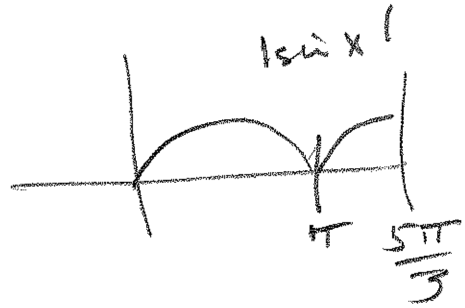
$$= -\cos\left(\frac{5\pi}{3}\right) - (-\cos(0))$$

$$= -\left(\frac{1}{2}\right) + 1 = \frac{1}{2}$$

10pts Bonus (10b)

$$\int_0^{\frac{5\pi}{3}} |\sin x \, dx| = \int_0^{\pi} \sin x \, dx - \int_{\pi}^{\frac{5\pi}{3}} \sin x \, dx$$

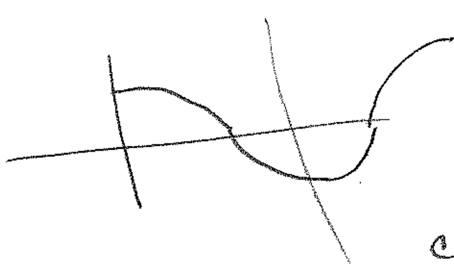




$$= -\cos x \Big]_0^{\pi} - (-\cos x) \Big]_{\pi}^{\frac{5\pi}{3}} = -\cos\pi - (-\cos 0)$$

$$+ \left[\cos\left(\frac{5\pi}{3}\right) - \cos\pi\right] = +1 + 1 + \frac{1}{2} - (-1)$$

$$= \boxed{\frac{7}{2}}$$



$\cos\pi = -1$

$-\cos\pi = +1$

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10 c 10 pts $\int x^3 (x^4+2)^5 \, dx$

$$= \frac{1}{4} \int (x^4+2)^5 (4x^3 \, dx) = \frac{1}{4} \frac{(x^4+2)^6}{6} + C$$

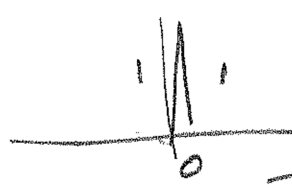
$$= \boxed{\frac{(x^4+2)^6}{24} + C}$$

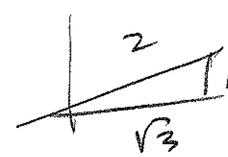
10 d 10 pts $\int_{\frac{1}{6}}^{\frac{1}{2}} \csc(\pi t) \cot(\pi t) \, dt$

$u = \pi t \Rightarrow du = \pi \, dt \rightarrow$ Need factor of π inside & $\frac{1}{\pi}$ outside

$u(\frac{1}{6}) = \frac{\pi}{6}$, $u(\frac{1}{2}) = \frac{\pi}{2}$

$$= \frac{1}{\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc u \cot u \, du$$





$$= \frac{1}{\pi} \Big[-\csc u \Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{\pi} \Big[-\csc \frac{\pi}{2} - \left(-\csc \frac{\pi}{6}\right) \Big]$$

$$= \frac{1}{\pi} \Big[-1 + 2 \Big] = \boxed{\frac{1}{\pi}}$$

(11) Arc Length $= L = \int_a^b \sqrt{1+f'(x)^2}\,dx$

$a=1,\ b=5$

$f(x) = 3x^2+2x$

$f'(x) = 6x+2$

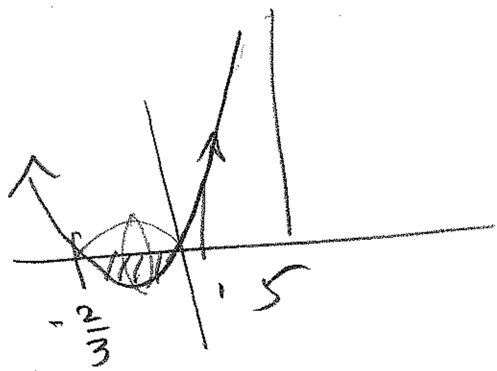
$f'(x)^2 = (6x+2)^2 = 36x^2+24x+4$

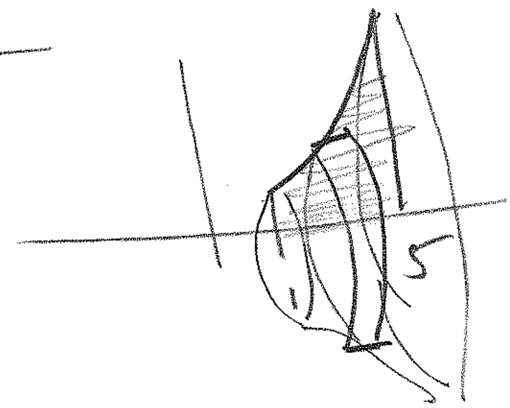
10 pts

$L = \int_1^5 \sqrt{1+(6x+2)^2}\,dx$

$3x^2+2x$
$= x(3x+2)$

12 @ 10 pts

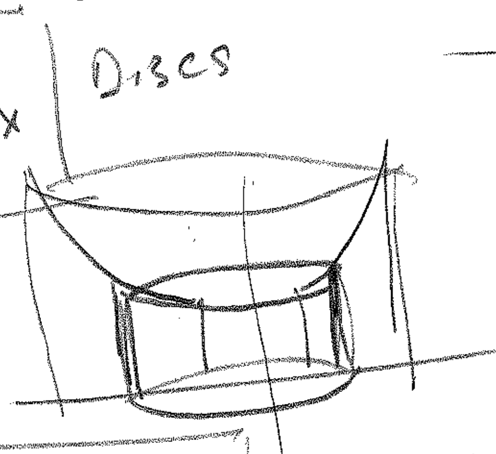




x-axis

$V = \pi \int_1^5 (3x^2+2x)^2\,dx$

Discs



12 b 10 pts

$$\boxed{V = 2\pi \int_1^5 x\,(3x^2+2x)\,dx}$$

Shells

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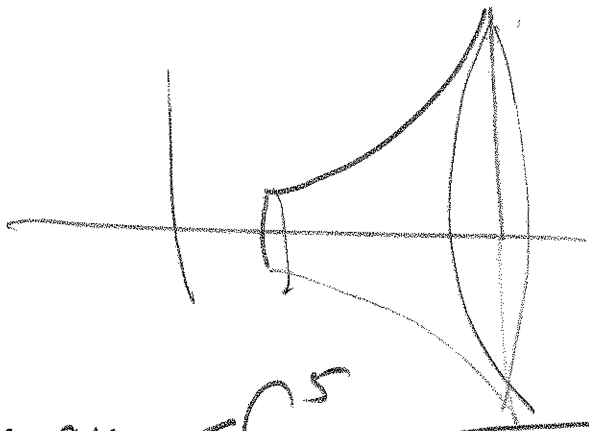
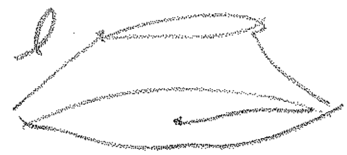
(13)



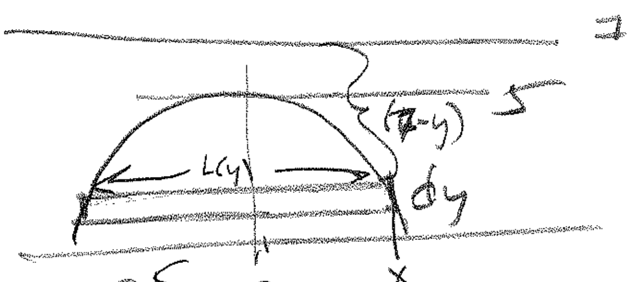


Surface area $\pi \int_1^5 f(x)\sqrt{1+f'(x)^2}\,dx$

10 pts

$$= 2\pi \int_1^5 (3x^2+2x)\sqrt{1+(6x+2)^2}\,dx$$

(14)



$x^2+y^2=5^2$

$x=\sqrt{25-y^2}$

$L(y)=2x$

$=2\sqrt{25-y^2}$

$$F = 62.4\int_0^5 (7-y)(2\sqrt{25-y^2})\,dy$$

10 pts