MAT 202 Final Test

100 Points (4 pts) Name_____

1. (4 pts) Solve $\ln(x+1)+\ln(x-1)=2$. Give an exact answer and an approximate answer.

2. (4 pts) Find the exact value of $\tan\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$.

3. Differentiate. *Do Not Simplify*.

   a. (4 pts) $g(x)=2^{5x}\cos(3x)$

   b. (4 pts) $V(t)=\arcsin\left(\arctan\sqrt{t}\right)$

   c. (4 pts) $y=\dfrac{\left(2x^2+1\right)^3}{(2x+1)^2(x-5)^6}$ (Use logarithmic differentiation.)

4. (4 pts) If $\tanh x = \frac{2}{3}$, find the value of the other 5 hyperbolic trigonometric functions. This should not require a calculator.

5. Evaluate the following:

   a. (4 pts) $\int x \sin x \, dx$

   b. (4 pts) $\int x \arcsin x \, dx$

c. (4 pts) $\int e^x \cos x \, dx$

d. (4 pts) $\int_2^3 \frac{x \, dx}{\sqrt{x^2 + 4x - 5}}$. (Hint: Complete the square under the radical.)

e. (4 pts) $\int_4^{\infty} x e^{-x^2} dx$

6. (4 pts) Write the integral for finding the arc length of $y = \sqrt{3 - x^2}, 0 \le x \le \frac{\sqrt{3}}{2}$. (You can check by noting this is part of a circle, but only if you have time at the end.)

7. (4 pts) Find an equation of the tangent to the curve described by the parametric equations $x = t^3 + t, y = t^4 + 1$ at $t = 2$.

8. Consider the curve in polar coordinates $r = 1 + 2\cos\theta$.

a. (5 pts) Check the curve in polar coordinates for symmetry and sketch its graph.

b. (5 pts) Find any value(s) of θ where the curve in #10 has a vertical tangent. Label this(these) point(s) in *rectangular coordinates* (rounded to 3 decimal places) on your graph in part #10, *or* do another sketch, below, that is more refined. The same sort of thing could be done for horizontal tangents, but don't waste time on that. (Hint: Recall $x = r\cos\theta$ and $r = 1 + 2\cos\theta$)

9. State whether the following series converge or diverge. If it converges, state whether the convergence is absolute or conditional. Justify your answer (Be clear on which test you are using), lest ye earn less than full credit.

a. (4 pts) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+17}}{25n^{2/3}}$

b. (4 pts) $\sum_{n=1}^{\infty} (-1)^n \frac{(1.1)^n}{n^4}$

c. (4 pts) $\sum_{n=4}^{\infty} \frac{3}{n^{3/4} - n^{1/4} - 1}$

d. (4 pts) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

10. (4 pts) Write the integral for the area of the surface obtained by rotating the curve

$$5y = x^2 + 15, \text{ for } 3 \le y \le 8$$

about the y-axis. *Do not evaluate the integral*.

11. Find a power series representation for each of the following:

a. (5 pts) $f(x) = \dfrac{1}{1+2x}$

b. (5 pts) $f(x) = x\sin\left(x^3\right)$

12. (4 pts) Estimate $\int_0^{1.3} x\sin\left(x^3\right)dx$ to three decimal places using power series.