MAT 202 Test 2
Fall, 2011 90 Points

Name KEY

Evaluate the following integrals. **Do your work on separate paper**. **Keep this cover sheet clean, except for your name**. Make sure everything is in order when you turn it in.

1. (9 pts) $\int x\sin(x)\,dx$

2. (9 pts) $\int e^x \sin(x)\,dx$ (Hint: This one is a classic. Unique (but not especially difficult) algebra involved.

3. (9 pts) $\int x\arctan(x)\,dx$ (Do it by parts, with $dv = x\,dx$ and $u = \arctan(x)$. Then make sure things are *proper* when you run up against the 2nd integral.)

4. (9 pts) $\int \frac{3x^2 - 17x + 26}{(x-2)(x-4)^2}\,dx$

5. (9 pts) $\int_0^3 \frac{x\,dx}{\sqrt{25 - x^2}}$ Special Instructions: Use *u*-substitution.

6. (9 pts) $\int_0^3 \frac{x\,dx}{\sqrt{25 - x^2}}$ Special Instructions: Use trigonometric substitution. 5 points bonus if you make the substitution for the limits of integration, as well, and work it all without a calculator.

7. (9 pts) $\int \sin(x)\cos(5x)\,dx$

8. (9 pts) $\int \tan(x)\sin^2(x)\,dx$

9. (9 pts) $\int_0^\infty \frac{dz}{z^2 + 4}$

10. (9 pts) $\int_3^7 \frac{dx}{\sqrt{x-3}}$

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KEY

(1) $\int x \sin x \, dx = I_1$ $\quad u = x \quad du = dx$

$dv = \sin x \, dx \quad v = -\cos x$

$= uv - \int v \, du = x(-\cos x) - \int -\cos x \, dx$

$$= \boxed{-x\cos x + \sin x + C}$$

(2) $\int e^x \sin x \, dx$ $\quad u = e^x \quad du = e^x dx$

$dv = \sin x \, dx \quad v = -\cos x$

$= uv - \int v \, du = e^x(-\cos x) - \int -\cos x \, e^x dx$

$$= -e^x \cos x + I_2$$

$I_2 = \int e^x \cos x \, dx$ $\quad u = e^x \quad du = e^x dx$

$dv = \cos x \, dx \quad v = \sin x$

$= uv - \int v \, du = e^x \sin x - \int e^x \sin x \, dx = e^x \sin x - I_1$

So, $I_1 = -e^x \cos x + I_2$

$I_1 = -e^x \cos x + e^x \sin x - I_1 \implies$

$2I_1 = -e^x \cos x + e^x \sin x \implies$

$$I_1 = \int e^x \sin x \, dx = \boxed{\frac{-e^x \cos x + e^x \sin x}{2} + C}$$

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(3) $\int x \arctan(x)\,dx$

$u = \arctan x \quad du = \dfrac{dx}{x^2+1}$

$dv = x\,dx \quad v = \frac{1}{2}x^2$

$= uv - \int v\,du = \frac{1}{2}x^2 \arctan(x) - \int \frac{1}{2}x^2 \cdot \dfrac{dx}{x^2+1}$

$= \frac{1}{2}x^2 \arctan(x) - I_1$

$I_1 = \frac{1}{2}\int \dfrac{x^2}{x^2+1}\,dx = \frac{1}{2}\int\left(1 - \dfrac{1}{x^2+1}\right)dx$

$$\begin{array}{r|l} & 1 \quad r = -1 \\ x^2+1 & x^2 + 0x + 0 \\ & -(x^2 \quad +1) \\ \hline & -1 \end{array}$$

$= \frac{1}{2}\int dx - \frac{1}{2}\int \dfrac{dx}{x^2+1}$

$= \frac{1}{2}x - \frac{1}{2}\arctan(x) + C$

This gives $\int x \arctan(x)\,dx = \boxed{\frac{1}{2}x^2 \arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C}$

(4) $\int \dfrac{3x^2 - 17x + 26}{(x-2)(x-4)^2}\,dx$

$\dfrac{3x^2 - 17x + 26}{(x-2)(x-4)^2} = \dfrac{A}{x-2} + \dfrac{B}{x-4} + \dfrac{C}{(x-4)^2}$

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$$3x^2-17x+26 = A(x-4)^2 + B(x-2)(x-4) + C(x-2)$$

$$= A(x^2-8x+16) + B(x^2-6x+8) + Cx-2C$$

$$= Ax^2-8Ax+16A+Bx^2-6Bx+8B+Cx-2C$$

$$\Longrightarrow A+B=3 \Longrightarrow A=3-B$$

$$-8A-6B+C=-17 \rightarrow -8(3-B)-6B+C=-17$$

and

$$-24+8B-6B+C=7$$

$$16A+8B-2C=26$$

$$2B+C=7$$

$$16(3-B)+8B-2C=26$$

$$48-16B+8B-2C=26$$

$$-8B-2C=-22$$

So $2B+C=7 \Longrightarrow$

$$C=7-2B$$

This gives $-8B-2C=-22$

$$-8B-2(7-2B)=-22$$

$$-8B-14+4B=-22$$

$$-4B=-8$$

$\boxed{B=2}$ so $A=3-B=\boxed{1=A}$

and $C=7-2B=7-4=\boxed{3=C}$

This gives

$$\int\frac{dx}{x-2}+\int\frac{2\,dx}{x-4}+\int\frac{3\,dx}{(x-4)^2}=\boxed{\ln|x-2|+\ln|x-4| + (-1)(x-4)^{-1}+C}$$

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(5) $$\int_0^3 \frac{x\,dx}{\sqrt{25-x^2}} = -\frac{1}{2}\int_0^3 \frac{-2x\,dx}{\sqrt{25-x^2}} = -\frac{1}{2}\cdot 2\sqrt{25-x^2}\Big]_0^3$$

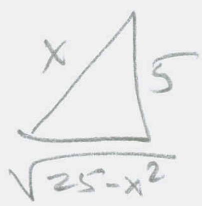
$$= -\sqrt{25-x^2}\Big]_0^3$$

$$= -\sqrt{25-3^2} - \left(-\sqrt{25-0^2}\right) = -\sqrt{16} + \sqrt{25} =$$

$$= -4 + 5 = \boxed{1}$$

(6) $$\int_0^3 \frac{x\,dx}{\sqrt{25-x^2}}$$

$x = 5\sin\theta \qquad dx = 5\cos\theta\, d\theta$



$$= \int_0^\beta \frac{5\sin\theta \cdot 5\cos\theta\, d\theta}{\sqrt{25 - 25\sin^2\theta}}$$

$x = 5\sin\theta = 0 \Longrightarrow \theta = 0$

$x = 5\sin\theta = 3 \Longrightarrow$

$\theta = \arcsin(3/5) = \beta$

$$= \int_0^\beta \frac{25\sin\theta\cos\theta\, d\theta}{5\sqrt{1-\sin^2\theta}} = 5\int_0^\beta \frac{\sin\theta\cos\theta\, d\theta}{\cos\theta}$$

$\cos\theta \geq 0$ on $[0, \beta]$

$$= 5\int_0^\beta \sin\theta\, d\theta = 5(-\cos\theta)\Big]_0^\beta$$

$$= -5\left[\cos\left(\arcsin\left(\tfrac{3}{5}\right)\right) - \cos(0)\right] = -5\left[\tfrac{4}{5} - 1\right]$$

$$= -5\left[-\tfrac{1}{5}\right] = \boxed{1}$$

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(7) $\int \sin(x)\cos(5x)\,dx$

$= \frac{1}{2}\int \left(\sin(-4x) + \sin(6x)\right) dx$

$= \frac{1}{2}\left[-\int \sin(4x)\,dx + \int \sin(6x)\,dx\right]$

$= \frac{1}{2}\left[-\left(-\frac{1}{4}\cos(4x)\right) + \left(-\frac{1}{6}\cos(6x)\right)\right] + C$

$= \boxed{\frac{1}{8}\cos(4x) - \frac{1}{6}\cos(6x) + C}$

(8) $\int \tan(x)\sin^2(x)\,dx = \int \tan(x)\left(1-\cos^2(x)\right) dx$

$= \int \tan(x)\,dx - \int \frac{\sin(x)}{\cos(x)} \cos^2(x)\,dx$

$u = \cos(x)$

$du = -\sin x\,dx$

$= -\ln|\cos(x)| - \int \sin(x)\cos(x)\,dx$ $\quad u = \sin(x)$, $du = \cos(x)\,dx$

$= \boxed{\ln|\sec(x)| - \frac{1}{2}\sin^2(x) + C}$

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(9) $\displaystyle\int_0^{\infty} \frac{dz}{z^2+4} = \lim_{t\to\infty} \int_0^{t} \frac{dz}{z^2+2^2} =$

$\displaystyle= \lim_{t\to\infty} \left[\frac{1}{2}\arctan\left(\frac{z}{2}\right) \right]_0^t$

$\displaystyle= \lim_{t\to\infty} \frac{1}{2}\arctan\left(\frac{t}{2}\right) - \frac{1}{2}\arctan(0)$

$\displaystyle= \frac{1}{2}\cdot 1 = \boxed{1}$

(10) $\displaystyle\int_3^{7} \frac{dx}{\sqrt{x-3}} = \int_0^{4} \frac{du}{\sqrt{u}} = \lim_{t\to 0} \left[2\sqrt{u} \right]_0^4$

$\displaystyle= 2\sqrt{4} - 2\sqrt{0} = \boxed{4}$