

1. (10 pts) Let $f(x) = \frac{2x+1}{4x-3}$. Find $f^{-1}(x)$, and state the domain and range of f and f^{-1} .

$$x = \frac{2y+1}{4y-3}$$

$$4yx - 3x = 2y + 1$$

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$$y(4x-2) = 3x+1$$

$$y = \frac{3x+1}{4x-2} = f^{-1}(x)$$

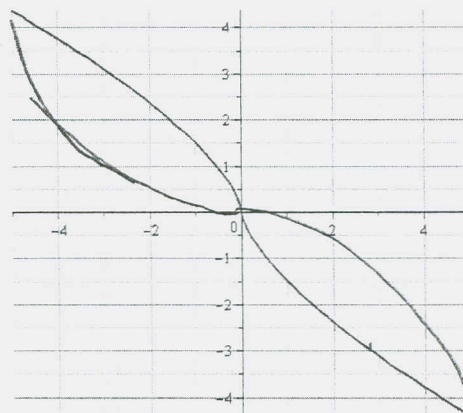
$$D(f) = R(f^{-1}) = \{x \mid x \neq \frac{3}{4}\}$$

$$D(f^{-1}) = R(f) = \{x \mid x \neq 2\}$$

2. The graph of f is given.

a. (5 pts) Estimate the value of $f^{-1}(-3) \approx 2.4$

- b. (5 pts) Sketch the graph of f^{-1} .



3. Find the exact value of each of the following:

a. (5 pts) $e^{\frac{1}{2}\ln(2)} = e^{\ln(\sqrt{2})}$

$$= \sqrt{2}$$



b. (5 pts) $\tan\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = 1$

4. (10 pts) Solve $e^x + 6e^{-x} = 5$ for x . Do not bother with a decimal approximation.

$$e^{2x} - 5e^x + 6 = 0$$

$$(e^x - 3)(e^x - 2) = 0$$

$$e^x = 3 \text{ or } e^x = 2$$

$$x = \ln(3) \text{ or } x = \ln(2)$$

5. Differentiate. Do not simplify.

a. (5 pts) $f(x) = \sqrt{1 + xe^{-3x}} = (1 + xe^{-3x})^{\frac{1}{2}} \Rightarrow$

$$f'(x) = \frac{1}{2}(1 + xe^{-3x})^{-\frac{1}{2}}(e^{-3x} - 3xe^{-3x})$$

b. (5 pts) $g(x) = (\cos(5x))^{x^2-2x}$

$$\ln(y) = (x^2 - 2x) \ln(\cos(5x))$$

$$\frac{y'}{y} = (2x - 2) \ln(\cos(5x)) - (x^2 - 2x) \cdot \frac{5 \sin(5x)}{\cos(5x)}$$

$$\Rightarrow y' = \left((2x - 2) \ln(\cos(5x)) - 5(x^2 - 2x) \tan(5x) \right) \cos(5x)^{x^2 - 2x}$$

c. (5 pts) $y = \ln\left(\frac{(x^2 + 1)^5 (x + 3)}{(2x + 1)^4}\right) = 5 \ln(x^2 + 1) + \ln(x + 3) - 4 \ln(2x + 1)$

$$\Rightarrow y' = 5 \cdot \frac{2x}{x^2 + 1} + \frac{1}{x + 3} - 4 \cdot \frac{2}{2x + 1}$$

6. (5 pts) Find the domain of $\ln(x^2 + 5x - 14)$.

$$x^2 + 5x - 14 > 0$$

$$(x + 7)(x - 2) > 0$$

$$\mathcal{D} = (-\infty, -7) \cup (2, \infty)$$



7. A population of bacteria triples in population every 10 hours.

a. (5 pts) Find the relative growth rate of the bacteria population.

Find k
 $t = \text{time in hrs.}$

$$P(t) = P_0 e^{kt}$$

$$P(10) = P_0 e^{10k} = 3P_0$$

$$e^{10k} = 3$$

$$10k = \ln(3)$$

$$\Rightarrow k = \frac{1}{10} \ln(3) \approx .1098612289$$

b. (5 pts) If the initial population was 100 cells, what is the bacteria population after 2 days?

$$P(t) = 100 e^{kt}$$

$$(2 \text{ days}) \left(\frac{24 \text{ hrs}}{1 \text{ day}} \right) = 48 \text{ hrs}$$

$$\Rightarrow P(48) = 100 e^{48k} = 100 e^{48 \cdot \frac{1}{10} \ln(3)} = 100 e^{\frac{24}{5} \ln(3)}$$

$$= 100 \left(e^{\ln(3)} \right)^{\frac{24}{5}} = 100 \cdot 3^{\frac{24}{5}} \approx 19,507 \text{ bacteria}$$

8. (5 pts) If $\sinh x = \frac{3}{7}$, find the value of the other 5 hyperbolic trigonometric functions.

This should not require a calculator.

$$\boxed{\operatorname{csch}(x) = \frac{7}{3}}$$

$$\boxed{\operatorname{sech}(x) = \frac{7}{\sqrt{58}} = \frac{7\sqrt{58}}{58}}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\cosh^2(x) = \sinh^2(x) + 1$$

$$\cosh^2(x) = \left(\frac{3}{7}\right)^2 + 1 = \frac{9}{49} + \frac{49}{49} = \frac{58}{49}$$

$$\Rightarrow \boxed{\cosh(x) = \frac{\sqrt{58}}{7}}$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$1 - \tanh^2(x) = \left(\frac{7}{\sqrt{58}}\right)^2$$

$$\tanh^2(x) = 1 - \frac{49}{58} = \frac{9}{58}$$

$$\boxed{\tanh(x) = \frac{3}{\sqrt{58}}}$$

$$\boxed{\operatorname{coth}(x) = \frac{\sqrt{58}}{3}}$$

9. (10 pts) Evaluate $\lim_{x \rightarrow 0} (\csc(x) - \cot(x))$. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)} \right)$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos(x)}{\sin(x)} \right) \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \left(\frac{\sin(x)}{\cos(x)} \right) = 0$$

10. Evaluate the integral.

a. (5 pts) $\int_0^1 3 \cdot 5^{-x^2} x dx$

$$= 3 \left(\frac{1}{2} \right) \int_0^1 5^u du = -\frac{3}{2} \cdot \frac{1}{\ln(5)} \left[5^u \right]_0^1 = \frac{-3}{2 \ln(5)} \left[\frac{1}{5} - 1 \right]$$

$$= -\frac{3}{2 \ln(5)} \left[-\frac{4}{5} \right] = \frac{6}{5 \ln(5)}$$

b. (5 pts) $\int \frac{\operatorname{sech}^2(x)}{\tanh(x) - 7} dx$

$$u = \tanh(x) - 7 \rightarrow du = \operatorname{sech}^2(x) dx$$

$$= \int \frac{du}{u} = \ln |u| + C = \ln |\tanh(x) - 7| + C$$

$du = \frac{1}{2} dx$ c. (5 pts) $\int \frac{1}{x\sqrt{x^2-4}} dx$

$$u = \frac{x}{2} \rightarrow x = 2u$$

$$= \int \frac{1 dx}{2u\sqrt{(2u)^2-4}} = \frac{1}{2} \int \frac{2 du}{u\sqrt{4u^2-4}} = \frac{1}{2} \int \frac{2 du}{2u\sqrt{u^2-1}} = \frac{1}{2} \int \frac{du}{u\sqrt{u^2-1}}$$

$$= \frac{1}{2} \sec^{-1}(u) + C = \frac{1}{2} \sec^{-1}\left(\frac{x}{2}\right) + C$$

(Hint: We talked about one like this in class on Wednesday. Ken made a nice suggestion for u that allowed us to factor a 4 out of the radical, and then it fit one of our cheat sheet formulas.)