MAT 202 Test 1
100 Points Name KEY

1. (10 pts) Let $f(x) = \dfrac{2x+1}{4x-3}$. Find $f^{-1}(x)$, and state the domain and range of f and f^{-1}.

$$\boxed{\begin{aligned} \mathcal{D}(f) &= \mathcal{R}(f^{-1}) = \{x \mid x \neq \tfrac{3}{4}\} \\ \mathcal{D}(f^{-1}) &= \mathcal{R}(f) = \{x \mid x \neq 2\} \end{aligned}}$$

$x = \dfrac{2y+1}{4y-3}$

$4yx - 3x = 2y + 1$

$4yx - 2y = 3x + 1$

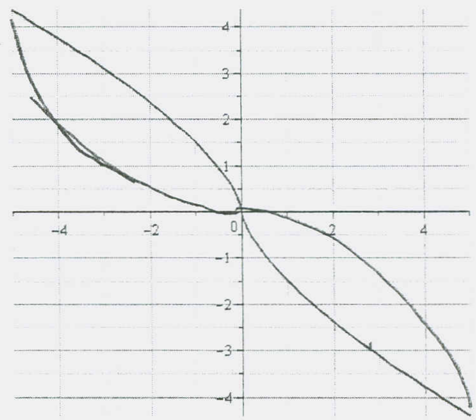
$y(4x-2) = 3x+1$

$y = \boxed{\dfrac{3x+1}{4x-2} = f^{-1}(x)}$

2. The graph of f is given.

   a. (5 pts) Estimate the value of $f^{-1}(-3)$. ≈ 2.4

   b. (5 pts) Sketch the graph of f^{-1}.

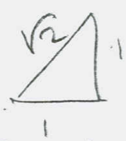


3. Find the exact value of each of the following:

   a. (5 pts) $e^{\frac{1}{2}\ln(2)} = e^{\ln(\sqrt{2})}$

   $= \sqrt{2}$

   b. (5 pts) $\tan\left(\cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)\right) = 1$



4. (10 pts) Solve $e^x + 6e^{-x} = 5$ for x. Do not bother with a decimal approximation.

$e^{2x} - 5e^x + 6 = 0$

$(e^x - 3)(e^x - 2) = 0$

$e^x = 3$ OR $e^x = 2$

$\boxed{x = \ln(3) \text{ OR } x = \ln(2)}$

5. Differentiate. Do not simplify.

a. (5 pts) $f(x)=\sqrt{1+xe^{-3x}}$ $= (1+xe^{-3x})^{\frac{1}{2}} \Rightarrow$

$$\boxed{f'(x) = \frac{1}{2}(1+xe^{-3x})(e^{-3x}-3xe^{-3x})}$$

b. (5 pts) $g(x)=(\cos(5x))^{x^2-2x}$

$$\ln(y) = (x^2-2x)\ln(\cos(5x))$$

$$\frac{y'}{y} = (2x-2)\ln(\cos(5x)) - (x^2 2x)\cdot\frac{5\sin(5x)}{\cos(5x)}$$

$$\Rightarrow \boxed{y' = \left((2x-2)\ln(\cos(5x)) - 5(x^2 2x)\tan(5x)\right)\cos(5x)^{x^2 2x}}$$

c. (5 pts) $y=\ln\left(\dfrac{(x^2+1)^5(x+3)}{(2x+1)^4}\right)$ $= 5\ln(x^2+1) + \ln(x+3) - 4\ln(2x+1)$

$$\Rightarrow \boxed{y' = 5\cdot\frac{2x}{x^2+1} + \frac{1}{x+3} - 4\cdot\frac{2}{2x+1}}$$

6. (5 pts) Find the domain of $\ln(x^2+5x-14)$.

$x^2+5x-14>0$

$(x+7)(x-2)>0$

$$\boxed{\mathcal{D} = (-\infty,-7)\cup(2,\infty)}$$

+ − +

−7 2

7. A population of bacteria triples in population every 10 hours.

a. (5 pts) Find the relative growth rate of the bacteria population.

Find K

t = time in hrs.

$P(t) = P_0 e^{kt}$

$P(10) = P_0 e^{10K} = 3P_0$

$e^{10K} = 3$

$10K = \ln(3)$

$10K = \ln(3) \Longrightarrow \boxed{K = \frac{1}{10}\ln(3)} \approx .109861229$

b. (5 pts) If the initial population was 100 cells, what is the bacteria population after 2 days?

$(2\text{ days})\left(\frac{24\text{ hrs}}{1\text{ day}}\right) = 48\text{hrs}$

$P(t) = 100e^{kt}$

$\Longrightarrow P(48) = 100e^{48K} = 100e^{48\cdot\frac{1}{10}\ln(3)} = 100e^{\frac{24}{5}\ln(3)}$

$= 100\left(e^{\ln(3)}\right)^{\frac{24}{5}} = 100\cdot 3^{\frac{24}{5}} \approx \boxed{19{,}507 \text{ bacteria}}$

8. (5 pts) If $\sinh x = \frac{3}{7}$, find the value of the other 5 hyperbolic trigonometric functions. This should not require a calculator.

$\boxed{\operatorname{csch}(x) = \frac{7}{3}}$

$\cosh^2(x) - \sinh^2(x) = 1$

$\cosh^2(x) = \sinh^2(x) + 1$

$\cosh^2(x) = \left(\frac{3}{7}\right)^2 + 1 = \frac{9}{49} + \frac{49}{49} = \frac{58}{49}$

$\Longrightarrow \boxed{\cosh(x) = \frac{\sqrt{58}}{7}}$

$\boxed{\operatorname{sech}(x) = \frac{7}{\sqrt{58}}} = \frac{7\sqrt{58}}{58}$

$1 - \tanh^2(x) = \operatorname{sech}^2(x)$

$1 - \tanh^2(x) = \left(\frac{7}{\sqrt{58}}\right)^2$

$\tanh^2(x) = -\frac{49}{58} + 1 = \frac{9}{58}$

$\boxed{\tanh(x) = \frac{3}{\sqrt{58}}} \longrightarrow \boxed{\coth(x) = \frac{\sqrt{58}}{3}}$

9. (10 pts) Evaluate $\lim_{x\to 0}(\csc(x)-\cot(x))$. $= \lim_{x\to 0}\left(\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}\right)$

$\infty - \infty$

$$= \lim_{x\to 0}\left(\frac{1-\cos(x)}{\sin(x)}\right) \overset{L'H}{=} \lim_{x\to 0}\left(\frac{\sin(x)}{\cos(x)}\right) = 0$$

$\frac{0}{0}$

10. Evaluate the integral.

$u = -x^2 \Rightarrow du = -2x\,dx$

$x=0 \Rightarrow u=0,\ x=1 \Rightarrow u=-1$

a. (5 pts) $\int_0^1 3\cdot 5^{-x^2}x\,dx$

$$= 3\left(-\tfrac{1}{2}\right)\int_0^{-1} 5^u\,du = -\frac{3}{2}\cdot\frac{1}{\ln(5)}\left[5^u\right]_0^{-1} = \frac{-3}{2\ln(5)}\left[\frac{1}{5}-1\right]$$

$$= -\frac{3}{2\ln(5)}\left[-\frac{4}{5}\right] = \frac{6}{5\ln(5)}$$

b. (5 pts) $\int \frac{\operatorname{sech}^2(x)}{\tanh(x)-7}dx$ $u = \tanh(x)-7 \Rightarrow du = \operatorname{sech}^2(x)\,dx$

$$= \int\frac{du}{u} = \ln|u| + C = \boxed{\ln|\tanh(x)-7| + C}$$

$du = \frac{1}{2}dx$ c. (5 pts) $\int \frac{1}{x\sqrt{x^2-4}}dx$

(Hint: We talked about one like this in class on Wednesday. Ken made a nice suggestion for u that allowed us to factor a 4 out of the radical, and then it fit one of our cheat sheet formulas.)

$u = \frac{x}{2} \Rightarrow x = 2u$

$$= \int\frac{1\,dx}{2u\sqrt{(2u)^2-4}} = \frac{1}{2}\int\frac{2\,du}{u\sqrt{4u^2-4}} = \frac{1}{2}\int\frac{2\,du}{2u\sqrt{u^2-1}} = \frac{1}{2}\int\frac{du}{u\sqrt{u^2-1}}$$

$$= \frac{1}{2}\sec^{-1}(u) + C = \boxed{\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C}$$