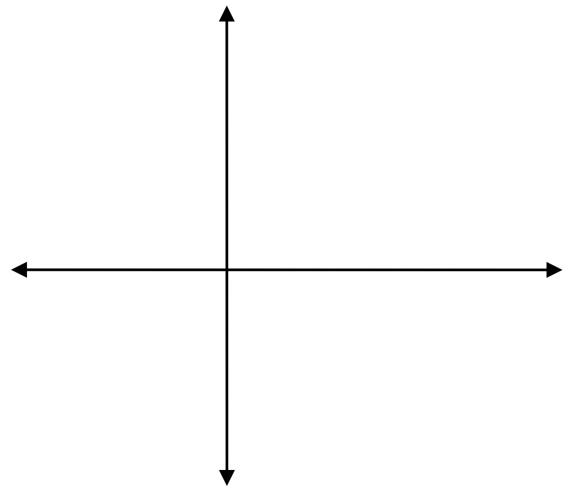
MAT 202 Test 4
100 Points Chapters 11, 12
Fall, 2009 Name___________________________

1. Let $x = 1 + t$ and $y = t^2 - 4t$.
   a. Sketch the curve by using the parametric equations to plot points, for $0 \leq t \leq 5$. Indicate with arrows the direction the curve is traced as t increases.

t	0	1	2	3	4	5
x						
y						



   b. Eliminate the parameter to find a Cartesian equation for the curve.

2. Suppose a curve is described by the parametric equations $x = t^4 + 1$, $y = t^3 + t$.

   a. Find an equation of the tangent to the curve at $t = -1$.

   b. Find the point(s) on the curve where the tangent is horizontal, if any.

   c. Find the point(s) on the curve where the tangent is vertical, if any.

3. Consider the curve given parametrically by
$x = \cos(t) + \cos(2t),\ y = \sin(t) + \sin(2t)$

a. Sketch the curve, using the table, below, as a guide.

t	0	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	2π
x							
y							

b. Find the tangent(s) to the curve at $(x, y) = (-1, 0)$. Add this (these) tangent line(s) to your sketch, above.

c. Write the integral for finding the arc length from $t = 0$ to $t = \pi$

4. Consider the curve described by the parametric equations $x = \sqrt{t},\ y = t^2 - 2t$.
Write the integral for finding the area bounded between the curve and the x-axis.

5. Check the function (in polar coordinates) $r = 1 + 2\sin\theta$ for symmetry and sketch its graph.

6. Convert each of the following to Cartesian form, and identify each graph as a common geometric figure.

a. $r = 3\cos\theta$

b. $r = \sec\theta\tan\theta$

7. Find the *exact* length of the curve $r = e^{2\theta}, 0 \le \theta \le \frac{\pi}{2}$.

8. (**S** 12.1) We have the following definition from lecture for $\lim_{n\to\infty} a_n = \infty$:

[D5] $a_n \xrightarrow{n\to\infty} \infty$ means if you give me $M > 0$, I can find $N \in \mathbb{N}$ such that $a_n > M$ for every $n > N$.

Consider the sequence $\{a_n\} = \left\{\frac{n+5}{\sqrt{n-1}}\right\}$. Find an N such that $a_n > 100$ for all $n > N$.

Bonus Prove that $\lim_{n\to\infty} \frac{n+5}{\sqrt{n-1}} = \infty$, in general. In other words, given any $M > 0$, find an N (as a function of M, such that $a_n > M$ for all $n > N$.

9. State whether the following series converge or diverge. Justify your answer, lest ye earn less than full credit.

a. $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt{n-1}}$

b. $\sum_{n=1}^{\infty} \frac{2}{n^2+1}$

c. $\sum_{n=1}^{\infty} \frac{2}{n^2-1}$

d. $\sum_{n=1}^{\infty} \frac{2}{n^2+1}$

e. $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

f. $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$

g. $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{\ln(n)}{n}$

h. $\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^{-n+1}$

i. $\sum_{n=1}^{\infty} \frac{5^n}{n!}$

j. $\sum_{n=1}^{\infty}\left(\frac{11n^3 - 1}{5n^3 + 7}\right)^n$

10. Find the exact value of $\sum_{n=1}^{\infty}12\left(\frac{3}{5}\right)^n$

11. Consider the series $S = \sum_{n=1}^{\infty}\frac{1}{n^{\frac{4}{3}}}$.

a. Use S_6 to estimate S. I will be docking points for roundoff errors.

b. Use an integral to obtain an upper bound on the error in using S_6.

c. Use *two* integrals and S_6 to obtain a *better* estimate for S.

d. Find N such that S_N is within 0.001 of S.

12.