1. We find arc length for $x = y + \sqrt{y}$, $1 \le y \le 2$. Simpson's Rule. $n = 10$.

$$\frac{dx}{dy} = 1 + \frac{1}{2\sqrt{y}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = 1 + \frac{1}{\sqrt{y}} + \frac{1}{4y} \Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 2 + \frac{1}{\sqrt{y}} + \frac{1}{4y}$$

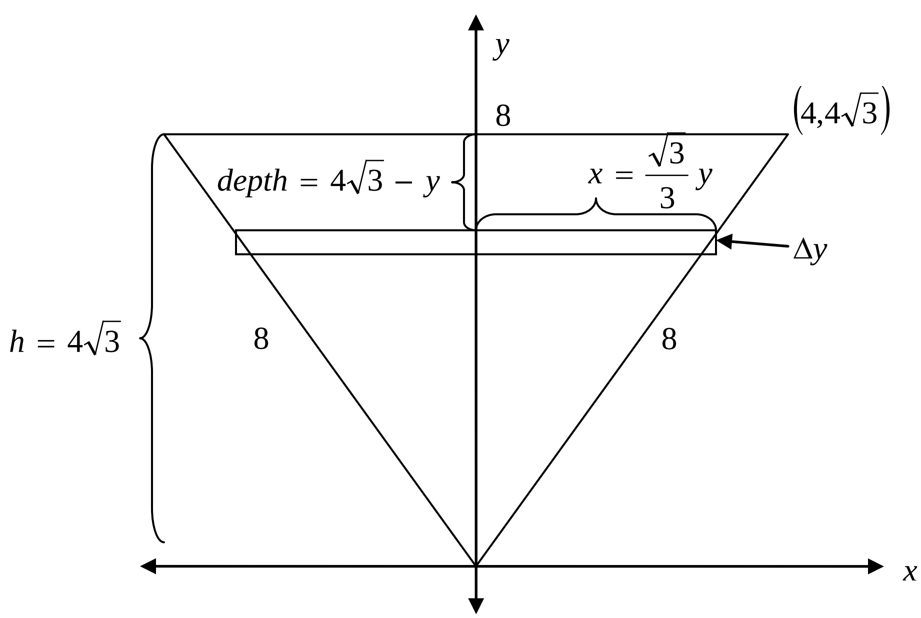
$\therefore L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy = \int_1^2 \sqrt{2 + \frac{1}{\sqrt{y}} + \frac{1}{4y}}\,dy \approx 1.732215081$, using Excel. Using Maple, Simpson's rule yields 1.732215082, which differs in the last digit. I'd lean toward Maple being more precise, because its purpose is to get its hands dirty on problems like this and Excel is a Microsoft product. :o)
As near as Maple can tell, the value is 1.732214672.

2. We find the surface area when $y = x + \sqrt{x}$ is revolved about the x-axis, from $x = 1$ to $x = 2$. To accomplish this, we evaluate the integral

$S = 2\pi \int_a^b y\,ds = \int_1^2 \left(x + \sqrt{x}\right)\sqrt{2 + \frac{1}{\sqrt{x}} + \frac{1}{4x}}\,dx$, with Simpson's Rule, $n = 10$.

When I used Maple with Simpson's Rule, I obtained 29.50656628.
When I used Excel, I obtained 29.50656629, again differing in the last digit, only.
As near as Maple can tell, the value is 29.50656808.

3. Force of Pressure on the end-wall of a trough with an equilateral triangular cross-section, with the dimensions shown in the diagram, below.



We implement the following integral:

$$
\begin{aligned}
P &= \rho g \int_0^{4\sqrt{3}} \left(4\sqrt{3} - y\right)\frac{\sqrt{3}}{3}\, y\, dy \\
&= \rho g \int_0^{4\sqrt{3}} \left(4y - \frac{\sqrt{3}}{3} y^2\right) dy \\
&= \rho g \left(2y^2 - \frac{\sqrt{3}}{9} y^3\right)\Bigg|_0^{4\sqrt{3}} \\
&= \rho g \left(2\left(4\sqrt{3}\right)^2 - \frac{\sqrt{3}}{9}\left(4\sqrt{3}\right)^3\right) \\
&= \rho g \left(2(16)(3) - \frac{\sqrt{3}}{9}(64)\left(3\sqrt{3}\right)\right) \\
&= \rho g \left(96 - \frac{9(64)}{9}\right) \\
&= \rho g (32) \\
&\approx 840(9.8)(32) \\
&= 263{,}424
\end{aligned}
$$

This gives HALF of the pressure, since the width I used was just the x-value associated with a given depth, $4\sqrt{3} - y$. So, the final answer is approximately 526,348 N.

Dimensional Analysis: $\rho = \frac{kg}{m^2}, g = \frac{m}{s^2}, \left(4\sqrt{3} - y\right) = m, \frac{\sqrt{3}}{3} y = m, dy = m$. This gives $\frac{kg}{m^2} \cdot \frac{m}{s^2} \cdot m^3 = \frac{kg \cdot m^2}{s^2} = N$, as we would hope and expect. :o)

4. We solve the logistic equation $y' = y\left(1 - \frac{y}{20}\right)$:

$$
\begin{aligned}
&\frac{dy}{dt} = y\left(\frac{20 - y}{20}\right) \\
&\Rightarrow \int \frac{20dy}{y(20 - y)} = \int dt \\
&\Rightarrow \int \left(\frac{-1}{y - 20} + \frac{1}{y}\right) dy = t + C \\
&\Rightarrow -\ln|y - 20| + \ln|y| = t + C \\
&\Rightarrow \ln\left|\frac{y}{y - 20}\right| = t + C
\end{aligned}
$$

For the purposes of the Logistic Model, we're assuming that $0 < y < 20$, with $y = 20$ being the carrying capacity of the environment. With this assumption, we see that $\dfrac{y}{y-20}$ *must* be negative, so that

$\left|\dfrac{y}{y-20}\right| = -\dfrac{y}{y-20} = e^{t+C} = e^t e^C = ke^t$, where $k = e^C$. Continuing this line of reasoning:

$$y = -ke^t(y-20) = -ke^t y + 20ke^t$$
$$\Rightarrow y + ke^t y = y(1+ke^t) = 20ke^t$$
$$\Rightarrow y = \frac{20ke^t}{1+ke^t} = \frac{1}{1+ke^{-t}}$$

This all changes when $y(0) = 25$, which leads to a sign change:

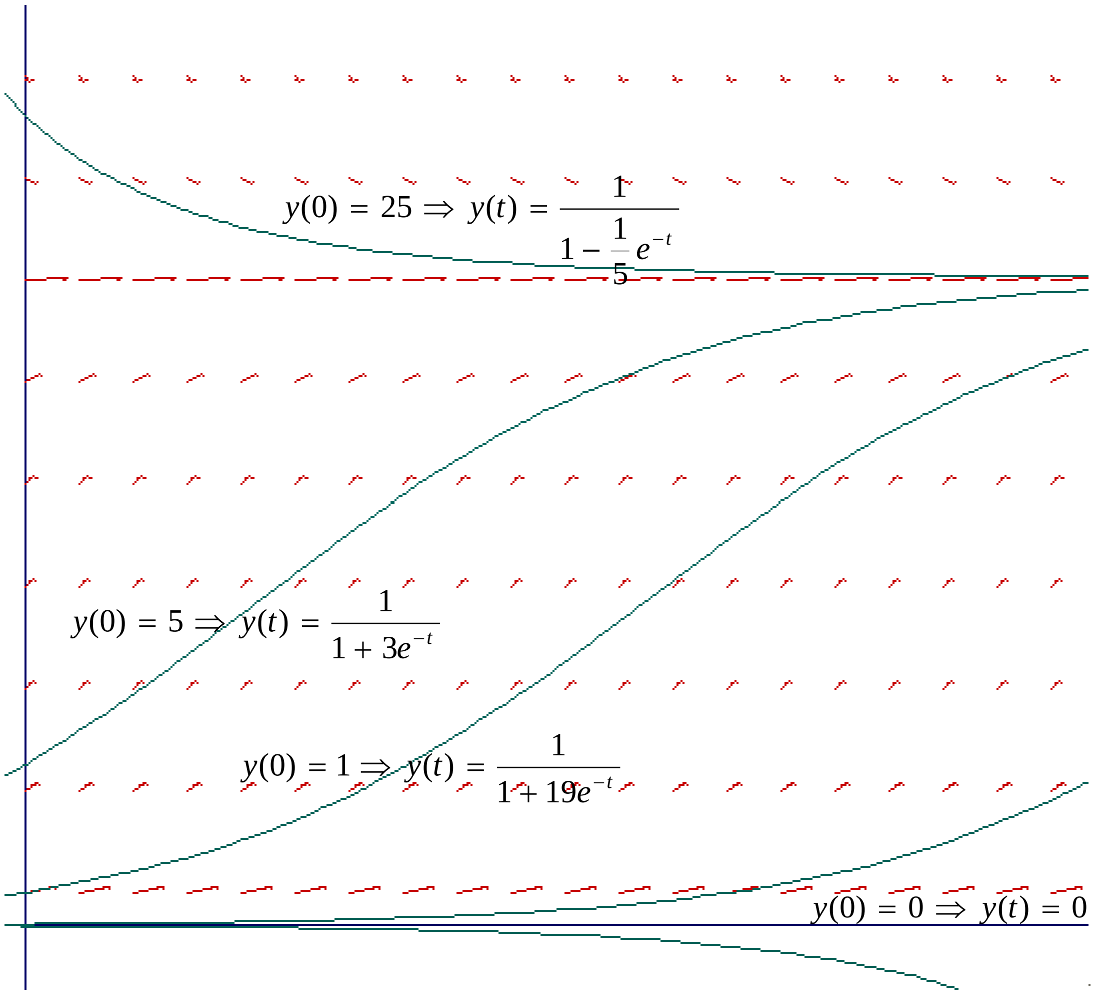
$\left|\dfrac{y}{y-20}\right| = \dfrac{y}{y-20} = e^{t+C} = e^t e^C = ke^t$, amounting to $y = \dfrac{1}{1-ke^{-t}}$. With the first version, we arrive at the conclusion that $k < 0$ in the $y(0) = 25$ case. A discerning student would say that this is ridiculous, considering the definition of k as e^C, which is positive. But with this little digression, we see that all is well.

$y(0) = 0 \Rightarrow y(t) = 0$. This degenerate case doesn't really fit the solution we found, since it's impossible to make $y(t) = 0$. But considering the differential equation, we see that y identically zero *is* as solution, and the direction field, below, confirms that the only solution containing $y(0) = 0$ is the null solution.

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{1+19e^{-t}}$$
$$y(0) = 5 \Rightarrow y(t) = \frac{1}{1+3e^{-t}}$$

$y(0) = 25 \Rightarrow y(t) = \dfrac{1}{1-\frac{1}{5}e^{-t}}$, which fits the *theory* of the differential equation, but *the mathematical theory doesn't fit reality*, since the model says that starting *above* the carrying capacity leads to a strictly decreasing solution that stays *above the carrying capacity*!!! In real life, this wouldn't happen. More like, there'd be catastrophic die-off, and, as in nature, the population would fluctuate in the vicinity of the carrying capacity.

Nature's creatures aren't blessed with an innate sense of balance. They're just geared to survive as long as possible and to have as many babies as possible... ;o)


$y(0) = 25 \Rightarrow y(t) = \frac{1}{1 - \frac{1}{5}e^{-t}}$
$y(0) = 5 \Rightarrow y(t) = \frac{1}{1 + 3e^{-t}}$
$y(0) = 1 \Rightarrow y(t) = \frac{1}{1 + 19e^{-t}}$
$y(0) = 0 \Rightarrow y(t) = 0$