MAT 121 Test 1

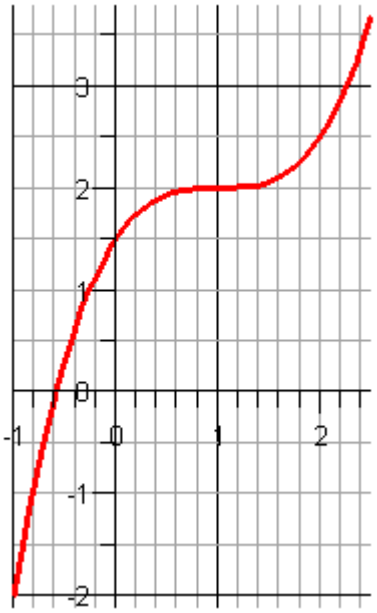
100 Points Name_____

1. Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or provide a counterexample.

   a. (4 pts) If f is one-to-one, with domain **R**, then $f^{-1}(f(6)) = 6$.

   b. (4 pts) If f is one-to-one and differentiable, with domain **R**, then $\left(f^{-1}\right)'(6) = \dfrac{1}{f'(6)}$.

2. The graph of g is given.



   a. (4 pts) Why is g one-to-one?

   b. (4 pts) Estimate the value of $g^{-1}(2)$.

   c. (4 pts) Sketch the graph of g^{-1}.

3. Find the exact value of each of the following:

   a. (4 pts) $\ln\left(e^{\pi}\right)$

   b. (4 pts) $\cos\left(\arctan\sqrt{3}\right)$

4. Solve the following equations for x.

   a. (4 pts) $\ln\left(1 + e^{-x}\right) = 3$

   b. (4 pts) $\ln(x + 1) + \ln(x - 1) = 1$

5. Differentiate.

a. (4 pts) $f(t) = t^2 \ln t$

b. (4 pts) $g(x) = 3^{mx} \cos(nx)$

c. (4 pts) $V(t) = \arctan\left(\arcsin \sqrt{t}\right)$

d. (4 pts) $y = \dfrac{\left(x^2 + 1\right)^4}{(2x+1)^3 (3x-1)^5}$ (Use logarithmic differentiation.)

e. (4 pts) $y = (\cos(3x))^{\tan(5x)}$

6. A bacterial culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour, the population has increased to 360 cells.

a. (4 pts) Find the number of bacteria after t hours.

b. (4 pts) Find the growth rate after 5 hours.

c. (4 pts) When will the population reach 10,000?

7. Cobalt-60 has a half-life of 5.24 years.

a. (4 pts) Find the mass that remains from a 100-mg sample after t years.

b. (4 pts) How long would it take for the mass to decay to 1 mg?

8. Evaluate the integral.

a. (4 pts) $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

b. (4 pts) $\int \frac{x}{\sqrt{1-x^4}}\,dx$

c. (4 pts) $\int \ln(\cos x)\tan x\,dx$

9. (4 pts) If $f(x) = x + x^2 + e^x$, find $\left(f^{-1}\right)'(1)$.

10. (4 pts) Find $f'(x)$ for $f(x) = \int_1^{\sqrt{x}} \frac{e^t}{t}\,dt$

11. (4 pts) If $\tanh x = \frac{3}{5}$, find the value of the other 5 hyperbolic trigonometric functions. This should not require a calculator.

12. (Bonus) (4 pts) Use mathematical induction to show that if $f(x) = xe^x$, then $f'(x) = (x + n)e^x$.