Do your work on separate paper. Do *not* put final answers on this cover sheet. Just circle them. Follow homework formatting guidelines for your work: One-sided, staple at top left corner. MARGIN at the top left corner. The only difference is I don't want you to fold them.

1. (10 pts) The function $f(x) = x^2 - 6x - 11$ is 1-to-1 on the restricted domain $[3, \infty)$. Find the inverse function. State its domain and range.

2. Find $\left(f^{-1}\right)'(5)$ for $f(x) = x^2 - 6x - 11$ $(x \geq 3)$, in two ways:

   a. (5 pts) Directly, using your answer from #1.

   b. (5 pts) Using our theorem for derivative of the inverse..

3. (5 pts each) Find the derivative with respect to x. Do not simplify.

   a. $y = 2 \cdot 3^{2x^2 - 3x}$

   b. $y = \ln\left(\dfrac{(x^2 - 2x)^3}{(2x+1)^5}\right)$ (Hint: Break it up into simpler logs!)

   c. $y = \log_3\left(x^2 - 2x\right)$

   d. $y = \left(x^2 - 3x\right)^{2x^2 + 5x}$

   e. $y = x^2 \sin^{-1}\left(x^2 - 3x\right)$

   f. $y = x^2 \tanh^{-1}\left(x^2 - 3x\right)$

4. (5 pts each) Evaluate the integral.

   a. $\displaystyle\int x\, e^{x^2 - 3x}\, dx$

   b. $\displaystyle\int \dfrac{dx}{x\sqrt{9 - x^2}}$

5. (5 pts each) Simplify:

   a. $\tan\left(\sec^{-1}(x)\right)$

   b. $\sin^{-1}\left(\sin\left(\dfrac{5\pi}{4}\right)\right)$

6. The half-life of Carbon-14 is about 5730 years. How old is a fire pit in which 30% of the original Carbon-14 remains?

7. Evaluate $\displaystyle\lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 3x}$ in two ways:

   a. Factor, cancel, pass to the limit.

   b. L'Hopital's rule

8. Evaluate the limits:

   a. $\displaystyle\lim_{x \to 0}\left(\dfrac{\sinh(x) - x}{x^3}\right)$

   b. $\displaystyle\lim_{x \to \infty}\left(x \sin\left(\dfrac{3}{x}\right)\right)$

   c. $\displaystyle\lim_{x \to \infty}(4x + 1)^{\cot(x)}$

202   TEST 1

① $f(x) = x^2 - 6x - 11$ $x \geq 3$

$y^2 - 6y - 11 = x$

$y^2 - 6y + 3^2 = x + 11 + 9$

$(y-3)^2 = x + 20$

$y - 3 = \pm \sqrt{x+20}$ TAKE TOP $\frac{1}{2}$!

$y = \boxed{\sqrt{x+20} + 3 = f^{-1}(x)}$

$\boxed{\begin{array}{c} \mathcal{D} = [-20, \infty) \\ R = [3, \infty) \end{array}}$

$x^2 - 6x - 11$

$= x^2 - 6x + 3^2 - 9 - 11$

$= (x-3)^2 - 20$

$\mathcal{D} = [3, \infty)$ by GIVEN

$R = [-20, \infty)$

② $(f^{-1})'(5)$ for $f(x) = x^2 - 6x - 11$

(a) $y = \sqrt{x+20} + 3$

$= (x+20)^{\frac{1}{2}} + 3$

$\Rightarrow y' = \frac{1}{2}(x+20)^{-\frac{1}{2}}(1)$

$\Rightarrow y'(5) = \frac{1}{2}(5+20)^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{25}} = \frac{1}{2} \cdot \frac{1}{5} = \boxed{\frac{1}{10} = (f^{-1})'(5)}$

(b) $(f^{-1})(5):$ $x^2 - 6x - 11 = 5$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ OR $x = -2$

$\longrightarrow f^{-1}(5)$

$f'(x) = 2x - 6$

$(f^{-1})'(5) = \dfrac{1}{f'(f^{-1}(5))}$

$= \dfrac{1}{2(8)-6} = \dfrac{1}{16-6}$

$= \boxed{\dfrac{1}{10} = (f^{-1})'(5)}$

③ ⓐ $y = 2 \cdot 3^{2x^2 - 3x}$ $\Longrightarrow$ $y' = 2 \cdot \ln(3) \cdot 3^{2x^2 - 3x} \cdot (4x - 3)$

$\frac{d}{du}\left[3^u\right] = \ln 3 \cdot 3^u$

ⓑ $y = \ln\left(\frac{(x^2 - 2x)^3}{(2x+1)^5}\right)$

$= 3\ln(x^2 - 2x) - 5\ln(2x - 1) \Longrightarrow$

$y' = 3 \cdot \frac{2x - 2}{x^2 - 2x} - 5 \cdot \frac{2}{2x - 1}$

ⓒ $y = \log_3(x^2 - 2x) \Longrightarrow$

$y' = \frac{1}{\ln(3)} \cdot \frac{2x - 2}{x^2 - 2x}$

ⓓ $y = (x^2 - 3x)^{2x^2 + 5x} \Longrightarrow$

$\ln y = (2x^2 + 5x)\ln(x^2 - 3x) \Longrightarrow$

$\frac{d}{dx}\left[\ln y\right] = \frac{y'}{y} = (4x + 5)\ln(x^2 - 3x) + (2x^2 + 5x)\left(\frac{2x - 3}{x^2 - 3x}\right)$

$y' = \left[(4x + 5)\ln(x^2 - 3x) + (2x^2 + 5x)\left(\frac{2x - 3}{x^2 - 3x}\right)\right](x^2 - 3x)^{2x^2 + 5x}$

③ cnt'd (e) $y = x^2 \sin^{-1}(x^2 - 3x)$ ⟶

$$y' = 2x \sin^{-1}(x^2 - 3x) + x^2 \cdot \frac{1}{\sqrt{1 - (x^2 - 3x)^2}} \cdot (2x - 3)$$

(f) $y = x^2 \tanh^{-1}(x^2 - 3x)$ ⟶

$$y' = 2x \tanh^{-1}(x^2 - 3x) + x^2 \cdot \frac{1}{\left(1 - (x^2 - 3x)^2\right)} \cdot (2x - 3)$$

④ (a) $\int (2x - 3) e^{x^2 - 3x} \, dx = \boxed{e^{x^2 - 3x} + C}$

(b) $\displaystyle\int \frac{dx}{x \sqrt{9 - x^2}} = \int \frac{dx}{x \sqrt{9\left(1 - \frac{x^2}{9}\right)}} = \int \frac{dx}{3x \sqrt{1 - \left(\frac{x}{3}\right)^2}}$
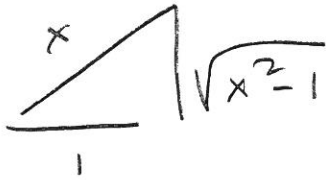
$u = \frac{x}{3}$ and $x = 3u$

$du = \frac{1}{3} dx$

So $dx = 3 du$

$= \displaystyle\int \frac{3 du}{3 \cdot 3u \sqrt{1 - u^2}} = \frac{1}{3} \int \frac{du}{u \sqrt{1 - u^2}} = -\frac{1}{3} \text{sech}^{-1}(u) + C$

$= \boxed{-\frac{1}{3} \text{sech}^{-1}\left(\frac{x}{3}\right) + C}$

5(a) $\tan\left(\sec^{-1}(x)\right) = \boxed{\sqrt{x^2-1}}$

(b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = \boxed{-\frac{\pi}{4}}$!

(6) $\frac{1}{2}$-life is 5730 yrs

$P_0 e^{5730k} = \frac{1}{2}P_0$

$e^{5730k} = \frac{1}{2}$

$5730k = \ln(1/2) = -\ln 2$

$$\boxed{k = -\frac{\ln 2}{5730}}$$

30% .3 left

$P_0 e^{kt} = .3P_0$

$e^{kt} = .3$

$kt = \ln(.3)$

$t = \frac{\ln(.3)}{k} = \frac{\ln(.3)\cdot 5730}{-\ln(2)} \approx 9952.812855$

$\approx \boxed{9953 \text{ yrs old}}$

202   TEST 1

⑦ ⓐ $\dfrac{x^2-x-6}{x^2-3x} = \dfrac{(x-3)(x+2)}{x(x-3)} = \dfrac{x+2}{x} \xrightarrow{x \to 3} \boxed{\dfrac{5}{3}}$

$x \neq 3$

ⓑ $\displaystyle\lim_{x \to 3} \dfrac{x^2-x-6}{x^2-3x} \;\; \dfrac{0}{0} = \lim_{x \to 3} \dfrac{2x-1}{2x-3} = \dfrac{2(3)-1}{2(3)-3} = \boxed{\dfrac{5}{3}}$

202   TEST 1

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a) $\lim\limits_{x \to 0} \dfrac{\sinh(x) - x}{x^3} \quad \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{\cosh(x) - 1}{3x^2} \quad \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{\sinh(x)}{6x}$

$\dfrac{0}{0}$ $\dfrac{0}{0}$ $\dfrac{0}{0}$

$\overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{\cosh(x)}{6} = \boxed{\dfrac{1}{6}}$

b) $\lim\limits_{x \to \infty} x \sin\left(\dfrac{3}{x}\right) = \lim\limits_{x \to \infty} \dfrac{\sin\left(\dfrac{3}{x}\right)}{\dfrac{1}{x}} \quad \overset{L'H}{=} \lim\limits_{x \to \infty} \dfrac{-\dfrac{3}{x^2}\cos\left(\dfrac{3}{x}\right)}{-\dfrac{1}{x^2}}$

$\infty \cdot 0$ $\dfrac{0}{0}$

$= \lim\limits_{x \to \infty} -3\cos\left(\dfrac{3}{x}\right) = \boxed{-3}$

cot x

x=0   x=π
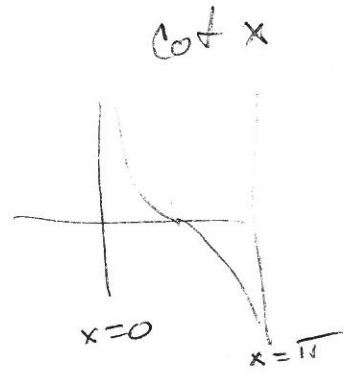
c) $\lim\limits_{x \to 0^+} (4x+1)^{\cot(x)}$

$\lim\limits_{x \to 0^+} \ln y = \lim\limits_{x \to 0^+} \cot(x) \ln(4x+1) = \lim\limits_{x \to 0^+} \dfrac{\ln(4x+1)}{\tan(x)}$

$\infty \cdot 0$ $\dfrac{0}{0}$

$\overset{L'H}{=} \lim\limits_{x \to 0^+} \dfrac{\dfrac{4}{4x+1}}{\sec^2 x} = \dfrac{\dfrac{4}{1}}{1} = \boxed{4}$