202 TEST 1

(1) $f(x) = x^2 - 6x - 11 \quad x \geq 3$

$y^2 - 6y - 11 = x$

$y^2 - 6y + 3^2 = x + 11 + 9$

$(y-3)^2 = x + 20$

$y - 3 = \pm\sqrt{x+20}$ TAKE TOP $\frac{1}{2}$!

$y = \boxed{\sqrt{x+20} + 3 = f^{-1}(x)}$

$\mathcal{D} = [-20, \infty)$

$\mathcal{R} = [3, \infty)$

$x^2 - 6x - 11$

$= x^2 - 6x + 3^2 - 9 - 11$

$= (x-3)^2 - 20$

$\mathcal{D} = [3, \infty)$ by GIVEN

$\mathcal{R} = [-20, \infty)$

(2) $(f^{-1})'(5)$ for $f(x) = x^2 - 6x - 11$

(a) $y = \sqrt{x+20} + 3$

$= (x+20)^{\frac{1}{2}} + 3$

$\Rightarrow y' = \frac{1}{2}(x+20)^{-\frac{1}{2}}(1)$

$\Rightarrow y'(5) = \frac{1}{2}(5+20)^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{25}} = \frac{1}{2} \cdot \frac{1}{5} = \boxed{\frac{1}{10} = (f^{-1})'(5)}$

(b) $(f^{-1})(5)$: $x^2 - 6x - 11 = 5$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ OR $x = -2$

$\rightarrow f^{-1}(5)$

$f'(x) = 2x - 6$

$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))}$

$= \frac{1}{2(8) - 6} = \frac{1}{16 - 6}$

$= \boxed{\frac{1}{10} = (f^{-1})'(5)}$

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(3) (a) $y = 2 \cdot 3^{2x^2-3x} \longrightarrow y' = \boxed{2 \cdot \ln(3) \cdot 3^{2x^2-3x} \cdot (4x-3)}$

$\frac{d}{du}\left[3^u\right] = \ln 3 \cdot 3^u$

(b) $y = \ln\left(\frac{(x^2-2x)^3}{(2x+1)^5}\right)$

$= 3\ln(x^2-2x) - 5\ln(2x-1) \longrightarrow$

$$\boxed{y' = 3 \cdot \frac{2x-2}{x^2-2x} - 5 \cdot \frac{2}{2x-1}}$$

(c) $y = \log_3(x^2-2x) \longrightarrow$

$$\boxed{y' = \frac{1}{\ln(3)} \cdot \frac{2x-2}{x^2-2x}}$$

(d) $y = (x^2-3x)^{2x^2+5x} \longrightarrow$

$\ln y = (2x^2+5x)\ln(x^2 3x) \longrightarrow$

$\frac{d}{dx}\left[\ln y\right] = \frac{y'}{y} = (4x+5)\ln(x^2 3x) + (2x^2+5x)\left(\frac{2x-3}{x^2 3x}\right)$

$\longrightarrow$

$$\boxed{y' = \left[(4x+5)\ln(x^2 3x) + (2x^2+5x)\left(\frac{2x-3}{x^2 3x}\right)\right](x^2-3x)^{2x^2+5x}}$$

202 TEST 1

3 cnt'd e $y = x^2 \sin^{-1}(x^2-3x) \longrightarrow$

$$\boxed{y' = 2x\sin^{-1}(x^2-3x) + x^2 \cdot \frac{1}{\sqrt{1-(x^2-3x)^2}} \cdot (2x-3)}$$

f $y = x^2\tanh^{-1}(x^2-3x) \longrightarrow$

$$\boxed{y' = 2x\tanh^{-1}(x^2-3x) + x^2 \cdot \frac{1}{(1-(x^2-3x)^2)} \cdot (2x-3)}$$

4 a $\int (2x-3)\, e^{x^2-3x}\, dx = \boxed{e^{x^2-3x} + C}$

b $$\int \frac{dx}{x\sqrt{9-x^2}} = \int \frac{dx}{x\sqrt{9(1-\frac{x^2}{9})}} = \int \frac{dx}{3x\sqrt{1-(\frac{x}{3})^2}}$$

$u = \frac{x}{3}$ and $x = 3u$

$du = \frac{1}{3}dx$

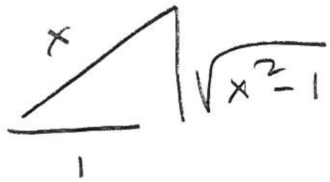
so $dx = 3du$

$$= \int \frac{3du}{3 \cdot 3u\sqrt{1-u^2}} = \frac{1}{3}\int \frac{du}{u\sqrt{1-u^2}} = -\frac{1}{3}\operatorname{sech}^{-1}(u) + C$$

$$= \boxed{-\frac{1}{3}\operatorname{sech}^{-1}\left(\frac{x}{3}\right) + C}$$

202 TEST 1

5 (a) $\tan(\sec^{-1}(x)) = \boxed{\sqrt{x^2-1}}$



(b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = \boxed{-\frac{\pi}{4}}$!

(6) $\frac{1}{2}$-life is 5730 yrs

$P_0 e^{5730k} = \frac{1}{2}P_0$

$e^{5730k} = \frac{1}{2}$

$5730k = \ln(1/2) = -\ln 2$

$\boxed{k = -\frac{\ln 2}{5730}}$

30% is left

$P_0 e^{kt} = .3P_0$

$e^{kt} = .3$

$kt = \ln(.3)$

$t = \frac{\ln(.3)}{k} = \frac{\ln(.3)\cdot 5730}{-\ln(2)} \approx 9952.812855$

$\approx$ $\boxed{\text{9953 yrs old}}$

202 TEST 1

7 a $\dfrac{x^2-x-6}{x^2-3x} = \dfrac{(x-3)(x+2)}{x(x-3)} = \dfrac{x+2}{x} \xrightarrow{x \to 3} \boxed{\dfrac{5}{3}}$

$x \neq 3$

b $\lim\limits_{x \to 3} \dfrac{x^2-x-6}{x^2-3x} = \lim\limits_{x \to 3} \dfrac{2x-1}{2x-3} = \dfrac{2(3)-1}{2(3)-3} = \boxed{\dfrac{5}{3}}$

$\dfrac{0}{0}$

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8

(a) $\lim_{x\to 0} \frac{\sinh(x) - x}{x^3} \overset{L'H}{=} \lim_{x\to 0} \frac{\cosh(x) - 1}{3x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{\sinh(x)}{6x}$

$\frac{0}{0}$ $\quad \frac{0}{0}$ $\quad \frac{0}{0}$

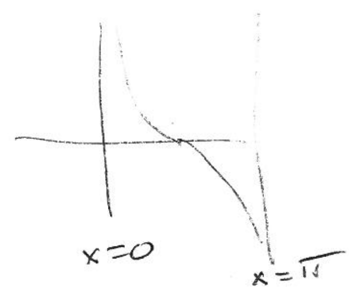
$\overset{L'H}{=} \lim_{x\to 0} \frac{\cosh(x)}{6} = \boxed{\frac{1}{6}}$

(b) $\lim_{x\to\infty} x \sin\left(\frac{3}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{3}{x}\right)}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{-\frac{3}{x^2}\cos\left(\frac{3}{x}\right)}{-\frac{1}{x^2}}$

$\infty \cdot 0$ $\quad \frac{0}{0}$

$= \lim_{x\to\infty} -3\cos\left(\frac{3}{x}\right) = \boxed{-3}$

cot x



(c) $\lim_{x\to 0^+} (4x+1)^{\cot(x)}$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \cot(x) \ln(4x+1) = \lim_{x\to 0^+} \frac{\ln(4x+1)}{\tan(x)}$

$\infty \cdot 0$ $\quad \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{4}{4x+1}}{\sec^2 x} = \frac{\frac{4}{1}}{1} = \boxed{4}$