1 $f^{(n)}(0) = (n+1)! \Rightarrow$

$$c_n = \frac{f^{(n)}(0)}{n!} = \frac{(n+1)!}{n!} = n+1$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} (n+1)x^n \quad (a)$$

(b) $$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+2)x^{n+1}}{(n+1)x^n}\right| = \frac{n+2}{n+1}|x| \xrightarrow{n\to\infty} |x| < 1 \Rightarrow \boxed{R=1}$$

2 $$f^{(n)}(7) = \frac{(-1)^n n!}{4^n(n+2)}$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n!}{4^n(n+2)n!} x^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(n+2)} x^n \text{ or } \sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n \frac{x^n}{n+2}$$

oops! $(x-7)^n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{4}|x| < 1 \Rightarrow |x| < 4 \Rightarrow R = 4$$

$|x-7| < 4$

3 $f(x) = 5xe^x$, $a=0$. Find 1st 4 nonzero terms

$$5x\sum_{n=0}^{\infty}\frac{x^n}{n!} = 5\sum_{n=0}^{\infty}\frac{x^{n+1}}{n!} = 5x + 5x^2 + \frac{5}{2}x^3 + \frac{5}{6}x^4$$

$0! \quad 1! \quad 2!$

Replace "+" with "," for the list.

Used e^x chart. Didn't use Taylor's formulation

4 Use Defn of Taylor series to find 1st 4 nonzero terms

0 $f(x) = \frac{4}{1+x} = 4(x+1)^{-1} = (-1)^0 \cdot 4 \cdot 0!\,(x+1)^{-1}$

1 $f'(x) = -4(x+1)^{-2} = (-1)^1 \cdot 4 \cdot 1!\,(x+1)^{-2}$ $\quad a=2$

2 $f''(x) = 8(x+1)^{-3} = (-1)^2(4)(2!)(x+1)^{-3}$

3 $f^{(3)}(x) = -24(x+1)^{-4} = (-1)^3(4)(3!)(x+1)^{-4}$

$\times$

0: $f(2) = \frac{4}{3}$

1: $f'(2) = \frac{-4}{3^2} = -\frac{4}{9}$

2: $f''(2) = 4(2)(3)^{-3} = \frac{8}{27}$

3: $f'''(2) = -4(3!)(3^{-4}) = \frac{-4(3!)}{3^4} = \frac{-4(3!)}{81}$

0: $\frac{\frac{4}{3}x^0}{0!} = \frac{4}{3}$

2: $\frac{4(2!)}{3^3} \cdot \frac{x^2}{2!} = \frac{4}{27}(x-2)^2$

1: $\frac{(-4/9)(x-2)}{1!} = -\frac{4}{9}(x-2)$

3: $-\frac{4(3!)}{3^4(3!)}(x-2)^3 = \frac{-4}{81}(x-2)^3$

⑤ $f(x) = 4\cos^2(x) = \frac{4(1+\cos(2x))}{2} = 2 + 2\cos(2x)$ $\quad a = 0$

n $\qquad$ 1st 4 nonzero

0 $\quad f(0) = 4$

1 $\quad f'(x) = -4\sin(2x)$, $f'(0) = 0$ $\qquad f^{(2n)}(0) =$

2 $\quad f''(x) = -8\cos(2x)$, $f''(0) = -8 = 4(-2)$

3 $\quad f'''(x) = 16\sin(2x)$ $f'''(0) = 0$

4 $\quad f^{(4)}(x) = 32\cos(2x)$ $\quad f^{(4)}(0) = 32 = (4)(8)$

$f^{(5)}(0) = 0$, $f^{(6)}(0) = -128 = 4(32)$

$4, \ \frac{-8}{2!}x^2, \ \frac{32}{4!}x^4, \ \frac{128}{6!}x^6$

$= 4, \ -4x^2, \ \frac{32}{4\cdot3\cdot2}x^4, \ \frac{128}{6\cdot5\cdot4\cdot3\cdot2}x^6 = \frac{3}{5\cdot3}x^6 = \frac{x^6}{5\cdot3}$ $\qquad \frac{\overset{8}{\cancel{\overset{16}{\cancel{\overset{64}{\cancel{128}}}}}}}{\underset{3}{\cancel{6}}\cdot5\cdot\cancel{4}\cdot3\cdot\cancel{2}} = \frac{8}{45}$

$4, -4x^2, \frac{4x^4}{3}, -\frac{8}{45}x^6$

E1 Approx $f(x) = \sqrt[3]{x}$ by Taylor Poly. of degree 2 @ $a=8$.

How accurate is it when $7 \le x \le 9$

$$T_2(x) = f(8) + \frac{f'(8)}{1!}(x-8) + \frac{f''(8)}{2!}(x-8)^2$$

$$|R_2(x)| \le \frac{M}{3!}|x-8|^3, \text{ where } |f'''(x)| \le M$$

$n = 2$, $n+1 = 3$

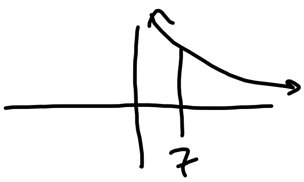
$$|f^{(n+1)}(x)| = |f^{(3)}(x)|$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

$$f''(x) = -\frac{2}{9}x^{-\frac{5}{3}}$$

$$f'''(x) = \frac{10}{27}x^{-\frac{8}{3}}$$

$$\frac{1}{x^{8/3}}$$



$x^{-\frac{8}{3}}$ is decreasing.
Its max on $[7,9]$ is
$7^{-\frac{8}{3}}$, so

$x^2 < x^{8/3} < x^3$
for $x > 1$

$$|R_2(x)| \le \frac{\frac{10}{27}\cdot 7^{-8/3}}{3!}|x-8|^3 < \frac{.0021}{3!} = .00035$$

$|x-8| \le 1$ on $[7,9]$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \Rightarrow$$

$$\sin\left(\frac{\pi x}{5}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{5} x\right)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{5^{2n+1}(2n+1)!}$$

$$\left(\frac{\pi}{5}\right)^{2n+1} = \frac{\pi^{2n+1}}{5^{2n+1}}$$

32. 0/2 points

Use the binomial series to expand the function as a power series.

$$5\left(1-\frac{x}{11}\right)^{2/3}$$

$$5-\frac{10}{33}x-10\sum_{n=2}^{\infty}\frac{1\cdot4\cdot7\cdot\cdots\cdot(3n-5)}{3^n n!}\left(\frac{x}{11}\right)^n$$

Algebra teaches

$$(1+x)^n=(x+1)^n$$

$$\sum_{k=0}^{n}\binom{n}{k}x^{n-k}$$

$$(x+y)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k$$

$$5\left(1+\left(-\frac{x}{11}\right)\right)^{2/3}$$

$$\cancel{5\sum_{k=0}^{\infty}\binom{n}{k}x^{n-k}}$$

$$5\sum_{n=0}^{\infty}\binom{k}{n}$$

$\ldots\quad \left|\frac{x}{11}\right|<1$

$|x|<11$

$\binom{k}{n}$ is way the book does these with $(1+x)^k$

$$=5\left[1+\frac{2}{3}\left(-\frac{x}{11}\right)+\frac{\frac{2}{3}\left(\frac{2}{3}-1\right)}{2!}\left(-\frac{x}{11}\right)^2+\frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{3!}\left(-\frac{x}{11}\right)^3+\cdots\right]$$

$$=5-\frac{10}{3}\cdot\frac{x}{11}-\frac{\frac{2}{9}\cdot5}{2!}\left(\frac{x}{11}\right)^2-\frac{\frac{8\cdot5}{27}}{3!}\left(\frac{x}{11}\right)^3-\cdots$$

$$=5-\frac{2}{3}\cdot5\cdot\frac{x}{11}-\frac{2(1)}{3^2}\cdot\frac{5}{2!}\left(\frac{x}{11}\right)^2-5\cdot\frac{2(1)(4)}{3!\,3^3}\left(\frac{x}{11}\right)^3+5\cdot\frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{4!\quad 3^4}\left(-\frac{x}{11}\right)^4$$

$+\ldots$

$$=\text{SAME}-\text{Last term}=\text{SAME}-\frac{2(1)(4)(7)}{4!\,3^4}\left(\frac{x}{11}\right)^4$$

$$-\ldots-\left(\frac{2}{n!}\right)\left(\frac{1\cdot4\cdot7\cdots(3n-5)}{3^n}\right)\left(\frac{x}{11}\right)^n\cdot5$$

$$=-10\left(\frac{1\cdot4\cdot7\cdots(3n-5)}{n!\,3^n}\right)\left(\frac{x}{11}\right)^n$$

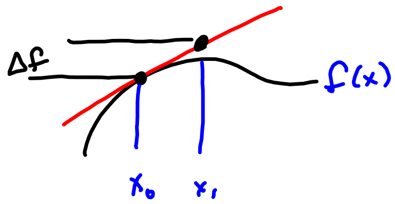
$$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{10\left(\frac{\cancel{1\cdot4\cdot7\cdots(3n-5)}(3(n+1)-5)}{(n+1)\cancel{!}\,3^{n+1}}\right)\left(\frac{x^{n+1}}{11^{n+1}}\right)}{10\left(\frac{\cancel{1\cdot4\cdots(3n-5)}}{\cancel{n!}\,3^n}\right)\frac{x^n}{11^n}}\right|=\left(\frac{3n-2}{(n+1)(3)}\right)\frac{|x|}{11}$$

$n\to\infty$ $\quad\frac{|x|}{11}$ want $<1\implies |x|<\boxed{11=R}$

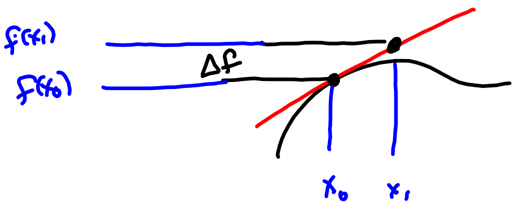
Questions?

31. A car is moving with speed 20 m/s and acceleration 2 m/s² at a given instant. Using a second-degree Taylor polynomial, estimate how far the car moves in the next second. Would it be reasonable to use this polynomial to estimate the distance traveled during the next minute?

You know f' & f''. Predict how far it'll move



$(x_0, f(x_0))$ to $(x_1, f(x_1))$, via tangent line.



$$f(x_1) = f(x_0+\Delta x) \approx f(x_0) + f'(x_0)(x-x_0)$$

$$f(x_0+\Delta x) - f(x_0) \approx f'(x_0)(x-x_0)$$

$$f(x_0+\Delta x) \approx f(x_0) + \frac{f'(x_0)(x-x_0)}{1!} + \frac{f''(x_0)(x-x_0)^2}{2!}$$

$$f(x_0+\Delta x) - f(x_0) \approx f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!}$$