Alternating Series Test If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots \qquad b_n > 0$$

satisfies

(i) $b_{n+1} \leqslant b_n$ for all n → on at last, eventually, long as so $n > 4$ or $n > 20$, as $\exists N \ni b_{n+1} \le b_n \ \forall n > N$

(ii) $\lim_{n \to \infty} b_n = 0$ → says quite a few b_{n+1}'s $< b_n$

then the series is convergent.

Test for Convergence/Divergence in #s 2 - 20.

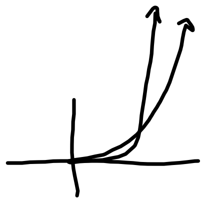
Text exercise examples

#4 $\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{\ln(n)}$ Converges

$\frac{1}{\ln(3)} - \frac{1}{\ln(4)} + \frac{1}{\ln(5)} - \ldots$

$\ln(x) \xrightarrow{x\to\infty} \infty$, so $b_n \xrightarrow{n\to\infty} 0$

#11 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$

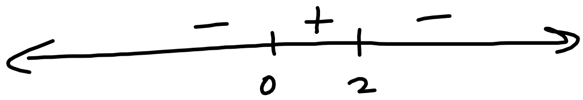


$b_n = \frac{n^2}{n^3+4}$

$f(x) = \frac{x^2}{x^3+4} \Longrightarrow f'(x) = \frac{2n(n^3+4) - n^2(3n^2)}{(n^3+4)^2}$

$\frac{f'g - fg'}{g^2} = \frac{2n^4 + 8n - 3n^4}{(n^3+4)^2} = \frac{-n^4 + 8n}{(n^3+4)^2} = \frac{-n(n^3-8)}{\sim}$

$= \frac{-n(n-2)(n^2+2n+4)}{(n^3+4)^2}$



$f' < 0 \ \forall x > 2$

so $b_{n+1} < b_n \ \forall n > 2$

Decreasing

$\frac{n^2}{n^3+4} \xrightarrow{n\to\infty} 0$ because $\frac{x^2}{x^3+4}$ is a proper rational function, with $y=0$ as H.A.

$\therefore$ it converges.

#14 $\sum_{n=1}^{\infty} (-1)^{n-1} \arctan(n)$ Diverges

show w/ a derivative.

arctangent is an increasing function that converges to $y = \frac{\pi}{2}$

show with graph?

Alternating Series Estimation Theorem If $s = \Sigma (-1)^{n-1} b_n$, where $b_n > 0$, is the sum of an alternating series that satisfies

$$\text{(i) } b_{n+1} \leqslant b_n \qquad \text{and} \qquad \text{(ii) } \lim_{n \to \infty} b_n = 0$$

Sweet.

then $$|R_n| = |s - s_n| \leqslant b_{n+1}$$

So $\sum_{k=1}^{n} (-1)^{k-1} b_k$ is close enough

we got $n > 5.2$, so
$n+1 = 6$
& $n = 5$.

#s 23-26 Show that the series is convergent. How many terms do we need in order to find the sum to the indicated accuracy?

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$ $\underbrace{(|\text{error}| < 0.00005)}_{\text{want}}$

$R_n \le b_{n+1} < .00005$

$\frac{1}{n^6} < .00005 = \frac{5}{100{,}000}$

$n^6 > \frac{100\,000}{5} = 20{,}000$

$\sqrt[6]{n^6} > \sqrt[6]{20{,}000}$

Estimate: $\sum_{n=1}^{5} \frac{(-1)^{n-1}}{n^6}$ ✓ because $b_6 <$ error

$b_6 = b_{n+1}$

25. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2 2^n}$ $(|\text{error}| < 0.0005)$

Need $\frac{1}{n^2 2^n} < .0005$

Looks like a technology exercise.

$n^2 2^n > 2000$

$\ln(n^2) + \ln(2^n) > \ln(2000)$

$2\ln(n) + n\ln(2) > \ln(2000)$

UGH

$solve(x^2 2^x = 2000)$

$$\frac{2\,\text{LambertW}(10\sqrt{5}\,\ln(2))}{\ln(2)}, \frac{2\,\text{LambertW}(-10\sqrt{5}\,\ln(2))}{\ln(2)}$$

$evalf(\%)$

5.862669048, 4.931129824 + 6.421829356 I

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WINDOW
 Xmin=0
 Xmax=10
 Xscl=.1
 Ymin=-100
 Ymax=2200
 Yscl=100
 Xres=1
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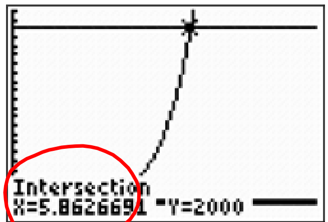

so b_6 will do it, & hence

S_5 will suffice.

27–30 Approximate the sum of the series correct to four decimal places.

27. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$

28. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$

29. $\sum_{n=1}^{\infty} (-1)^n n e^{-2n}$

30. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\, 4^n}$

4 decimal-place accuracy means
Error < .00005