EXAMPLE 8
§ 11.2 #35

Questions about 11.1 or 11.2?

$\boxed{T6}$ $\left(\sum a_n \longrightarrow\right) \Rightarrow \left(a_n \xrightarrow{n\to\infty} 0\right)$

" $\not\longrightarrow$ " Diverges

" $\longrightarrow$ " Converges.

The converse of T6 does NOT HOLD.

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} \not\longrightarrow \text{ although } \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

$$\frac{1}{n^{\frac{1}{2}}} \xrightarrow{n\to\infty} 0$$

$\boxed{E}$ Find the sum:

$$\sum\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right)$$

$\sum \frac{3}{n(n+1)}$ converges by comparison to $\sum \frac{3}{n^2}$

Integral Test
(Direct) Comparison Test* is old-school/grad-school/Advanced Calculus.
Limit Comparison Test is more flexible

$$\frac{3}{n(n+1)} = \frac{3}{n^2+n} < \frac{3}{n^2} \text{ \& } \sum_{n=1}^{\infty} \frac{3}{n^2} = 3\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ converges, by P-Test.}$$

Not bad. $\frac{3}{n(n-1)}$ is harder.

$$= \frac{3}{n^2-n} < \frac{3}{n^2} \quad \text{No!}$$

$>$! I made the denominator bigger!
So $\frac{3}{n^2}$ is SMALLER FRACTION.

But & here's the trick

$$\frac{3}{n^2-n} < \frac{3}{n^2-\frac{1}{2}n^2} \text{ for } n \text{ sufficiently large.}$$

$$\text{\& } \frac{3}{\frac{1}{2}n^2} = \frac{6}{n^2} \text{ passes the P-Test. } 6\sum \frac{1}{n^2} \longrightarrow$$

$$\frac{n^2}{2} - n > 0, \text{ eventually}$$

Recall

$$\int_1^{\infty} \frac{dx}{x^p} \text{ converges if } p > 1. \text{ (Diverges, otherwise)}$$

E Find the sum:

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{2^n}\right)$$

$$\frac{3}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} = \frac{3}{n} - \frac{3}{n+1}$$

$$3 = A(n+1) + Bn$$

$n=0 \quad 3 = A$

$n=-1 \quad 3 = -B$

$$\sum_{k=1}^{n} \frac{3}{k(k+1)} = 3\left[\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \dots + \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}} + \cancel{\frac{1}{n}} - \frac{1}{n+1}\right] = 3\left[1 - \frac{1}{n+1}\right] \xrightarrow{n\to\infty} 3$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n} + \dots$$

$$= \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2^2} + \dots + \frac{1}{2}\cdot\frac{1}{2^{n-1}} + \dots$$

$= \text{Geometric Series } \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1} = \sum_{n=1}^{\infty} a r^{n-1} = \frac{a}{1-r}, \; a=\frac{1}{2}, r=\frac{1}{2}$

So it equals $\frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1 \Longrightarrow$

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right) = 3 + 1 = \boxed{4 = 5}$$

The series of the sum is the sum of the series, if the summands converge, separately.

$A + B$

summands or terms

For i=1 thru n do(~)
For j=1 thru n do(~)

$$\int_a^b f(t)\,dt = \int_a^b f(u)\,du$$

Write $.\bar{8}$ as a fraction (ratio of integers)

$$x = .8888\ldots \qquad x = 5.\overline{123}123\overline{123}$$
$$10x = 8.8888\ldots \qquad 1000x = 5123.123123123$$
$$9x = 8 \Rightarrow x = \frac{8}{9} \qquad 999x = 5118 \Rightarrow x = \frac{5118}{999} = \frac{1706}{333}$$

$$.8888\ldots = 8\cdot\frac{1}{10} + 8\cdot\frac{1}{10^2} + 8\cdot\frac{1}{10^3} + 8\cdot\frac{1}{10^4} + \cdots$$

a

$a = 8,\ r =$ Nahhhh

$$8\cdot\frac{1}{10} + \left(8\cdot\frac{1}{10}\right)\left(\frac{1}{10}\right) + \left(8\cdot\frac{1}{10}\right)\left(\frac{1}{10^2}\right) + \cdots$$

$$= \frac{4}{5} + \frac{4}{5}\cdot\frac{1}{10} + \frac{4}{5}\cdot\frac{1}{100} + \cdots$$

$a = \frac{4}{5},\ r = \frac{1}{10}$ & we have $\sum_{n=1}^{\infty}\left(\frac{4}{5}\right)\left(\frac{1}{10}\right)^{n-1}$

$$= \frac{\frac{4}{5}}{1-\frac{1}{10}} = \frac{\frac{4}{5}}{\frac{9}{10}} = \frac{4}{5}\cdot\frac{10}{9} = \frac{8}{9}$$

$$5.123123123\ldots = 5 + .123123123$$
$$= 5 + (123)\left(\frac{1}{1000}\right) + (123)\left(\frac{1}{10^6}\right) + (123)\left(\frac{1}{10^9}\right)$$

3−1

$$= 5 + \underbrace{(123)\left(\frac{1}{10^3}\right)}_{n=1} + \underbrace{(123)\left(\frac{1}{10^3}\right)\left(\frac{1}{10^3}\right)^1}_{n=2} + \underbrace{(123)\left(\frac{1}{10^3}\right)\left(\frac{1}{10^3}\right)^2}_{n=3}$$
$$+ \cdots + \left(\frac{123}{10^3}\right)\left(\frac{1}{10^3}\right)^{n-1} + \cdots$$

$$= 5 + \sum_{k=1}^{\infty}\left(\frac{123}{10^3}\right)\left(\frac{1}{1000}\right)^{n-1} = 5 + \frac{\frac{123}{1000}}{1-\frac{1}{1000}} = \frac{\frac{123}{1000}}{\frac{999}{1000}}$$

$$= 5 + \frac{123}{1000}\cdot\frac{1000}{999} = \frac{123}{999} + 5 = \frac{41}{333} + 5,\ \text{etc.}$$

57–63 Find the values of x for which the series converges. Find the sum of the series for those values of x.

57. $\sum_{n=1}^{\infty} (-5)^n x^n$ **58.** $\sum_{n=1}^{\infty} (x+2)^n$

59. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$ **60.** $\sum_{n=0}^{\infty} (-4)^n (x-5)^n$

61. $\sum_{n=0}^{\infty} \frac{2^n}{x^n} = \sum_{n=1}^{\infty} \left(\frac{2}{x}\right)^{n-1}$ **62.** $\sum_{n=0}^{\infty} \frac{\sin^n x}{3^n}$

63. $\sum_{n=0}^{\infty} e^{nx}$

57) $\sum_{n=1}^{\infty} (-5)^n x^n$

$= \sum_{n=1}^{\infty} (-1)^n 5^n x^n$

$= \sum_{n=1}^{\infty} (-1)^n (5x)^n$

$= \sum_{n=1}^{\infty} (-5x)^n = \sum_{n=1}^{\infty} (-5x)(-5x)^{n-1}$

$= \frac{-5x}{1-(-5x)} = \boxed{\frac{-5x}{5x+1} = \text{Sum}}$

Need: $-1 < -5x < 1$

$\frac{1}{5} > x > -\frac{1}{5}$

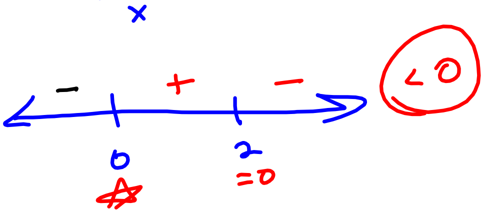
#61 Want/Need

$-1 < \frac{2}{x} < 1$

$\frac{2}{x} < 1$

$\frac{2}{x} - 1 < 0$

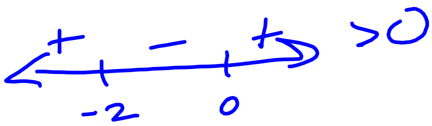
$\frac{2-x}{x} < 0$



$\frac{2}{x} > -1$

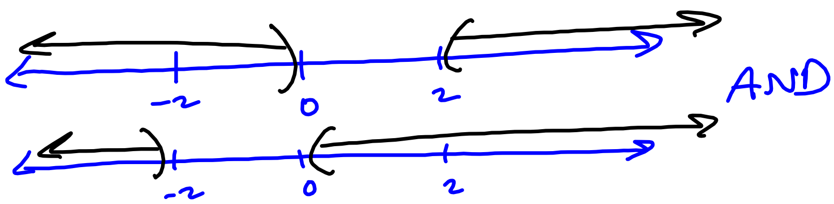
$\frac{2}{x} + 1 > 0$

$\frac{2+x}{x} > 0$

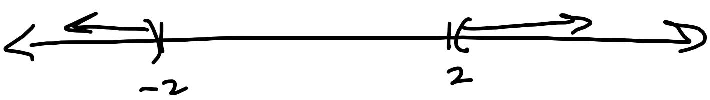


$x < -2$ OR $x > 0$

$x < 0$ OR $x > 2$

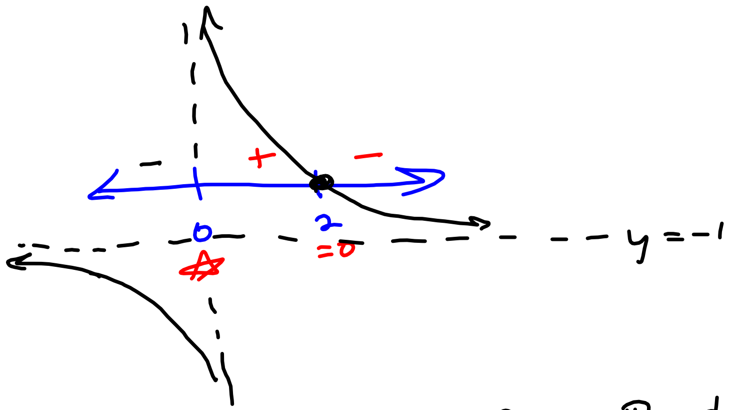


Combine:



$(-\infty, -2) \cup (2, \infty)$

$\frac{2-x}{x}$ $y=-1$ is H.A.



Limit Comparison If $c \in \mathbb{R}$ & $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, then $\sum_{n=1}^{\infty} a_n$ & $\sum_{n=1}^{\infty} b_n$ have the same convergence properties.

This is <u>much</u> easier than <u>Direct</u> Comparison.

Show that $\sum_{n=1}^{\infty} \frac{n^3+7n^2-5n+11}{4n^5-3n^3+2n-5}$ converges.

Looks like $\frac{1}{n^2}$, sort of $\frac{\deg(\text{top})}{\deg(\text{bottom})} = \frac{3}{5}$, so it's like $\frac{m}{n^2}$

$$\frac{\frac{n^3+7n^2-5n+11}{4n^5-3n^3+2n-5}}{\frac{1}{n^2}}$$

$$= \frac{(n^3+\text{loower})(n^2)}{(4n^5+\text{lower})} = \frac{n^5+\text{lower}}{4n^5+\text{lower}} \xrightarrow{n\to\infty} \frac{1}{4} \in \mathbb{R}$$

In detail: $\frac{n^5+7n^4-5n^3+11n^2}{4n^5-3n^3+2n-5} = \frac{\cancel{n^5}\left(1+\frac{7}{n}-\frac{5}{n^2}+\frac{11}{n^3}\right)}{\cancel{n^5}\left(4-\frac{3}{n^2}+\frac{2}{n^4}-\frac{5}{n^5}\right)}$

$\xrightarrow{n\to\infty} \frac{1}{4}$