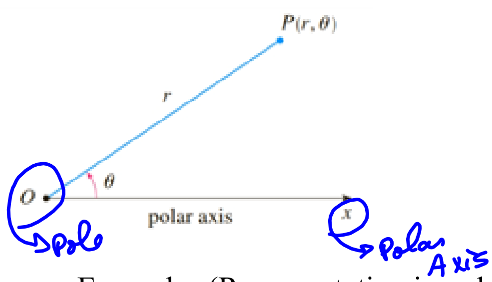
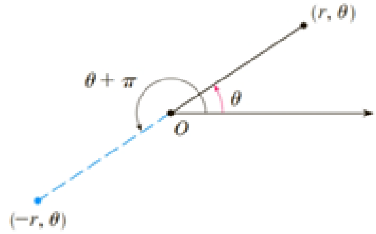
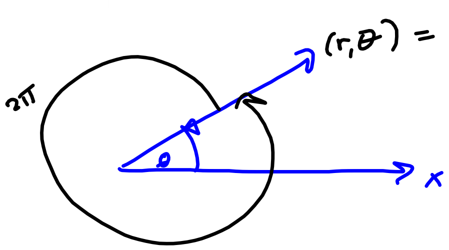
## Section 10.3 Polar Coordinates

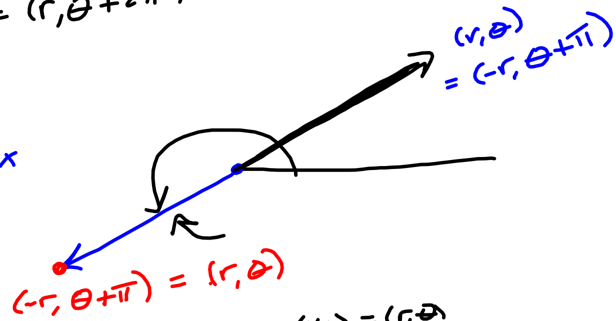



Examples (Representation in polar coordinates is not unique)

$(r,\theta) = (r, \theta + 2n\pi)$ and $\boxed{(-r, \theta + (\cancel{2n}+1)\pi)}$ $\therefore$



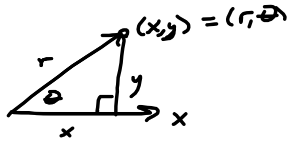




Recall $\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

similarly, $y = r\sin\theta$

& $\boxed{r^2 = x^2 + y^2 \text{ \& } \frac{y}{x} = \tan\theta}$ → More than one $\theta$ possible

Make sure you're in the right quadrant; arctangent only "sees" QI & QIV.



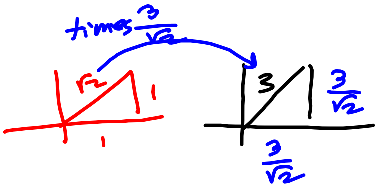

E Convert $(3, \frac{\pi}{4})$ to Cartesian coords

$r = 3, \theta = \frac{\pi}{4}$

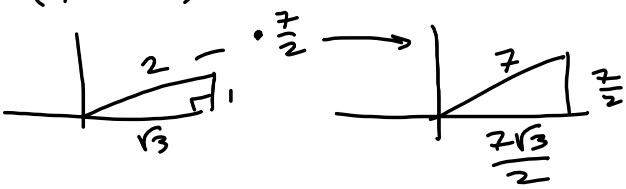
$x = 3\cos\theta$
$= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$y = 3\sin\theta = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$

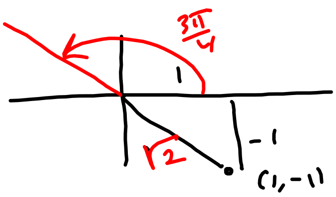



$(r,\theta) = (7, \frac{\pi}{6})$



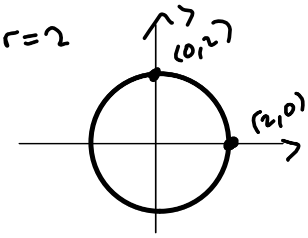

$(x,y) = (\frac{7\sqrt{3}}{2}, \frac{7}{2})$

E Convert $(1,-1)$ to Polar coords.



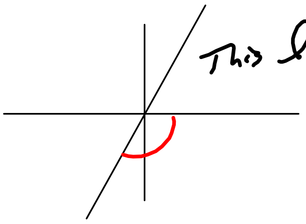

$(r,\theta) = (\sqrt{2}, -\frac{\pi}{4}) = (\sqrt{2}, \frac{7\pi}{4})$
$= (-\sqrt{2}, \frac{3\pi}{4})$

E $r=2$




circle of radius 2!

E $\theta = -2 = -2\left(\frac{180^\circ}{\pi}\right) \approx -115^\circ$



This line!

$\tan\theta = \frac{y}{x} = \text{slope}$

$\tan(-2) = \frac{y}{x}$

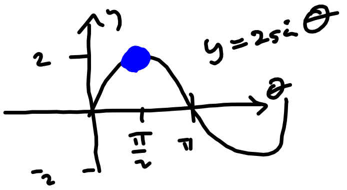
$\cancel{-2 = \arctan\left(\frac{y}{x}\right)}$

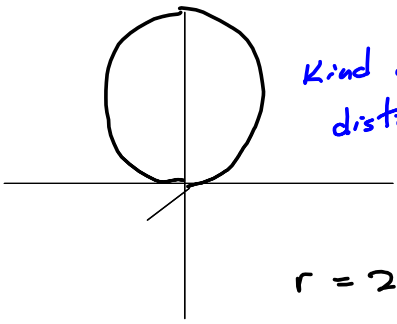
$y = \tan(-2)\,x$

$\approx 2.185\,x$

E Sketch $r = 2\sin\theta$

Find Cartesian representation



Kind of OK, but distorted a bit.

$r = 2\sin\theta = 2\cdot\frac{y}{r}$

$\Longrightarrow r^2 = 2y$

$x^2+y^2 = 2y$

$x^2+y^2-2y+1^2 = 0+1$

$x^2+(y-1)^2 = 1$

is circle of radius 1 centered @ $(0,1)$

$x^2 = (x-0)^2$

Think of r as $r = f(\theta)$

$x = r\cos\theta, \; y = r\sin\theta$

$x = f(\theta)\cos\theta \; , \; y = f(\theta)\sin\theta \longrightarrow$

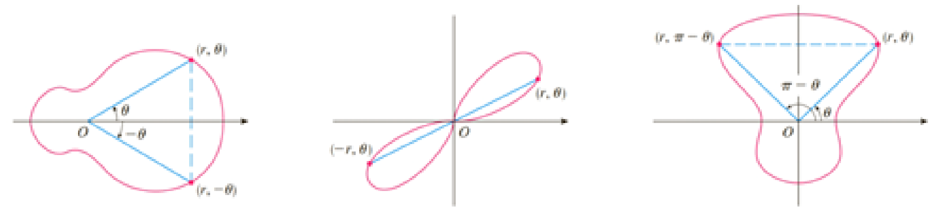
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta + f(\theta)(-\sin\theta)}$$

$$= \frac{f'(\theta)\sin\theta + r\cos\theta}{f'(\theta)\cos\theta - r\sin\theta} \quad \overset{\text{SET}}{=} 0 \longrightarrow$$

$f'(\theta)\sin\theta + r\cos\theta = 0 = \frac{dy}{d\theta}$ Horizontal Tangents

$f'(\theta)\cos\theta - r\sin\theta = 0 = \frac{dx}{d\theta}$ Vertical Tangents

Symmetry



(a) (b) (c)

symmetry about polar axis

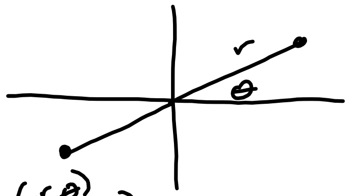
$r = \cos\theta$

$r = \cos(-\theta) = \cos(\theta)$

SAME!

Symmetric about Polar Axis

Pole:




$(-r, \theta)$

$(r, \theta + \pi)$

check: $r = 2\cos\theta$

$-r = 2\cos\theta$ ?

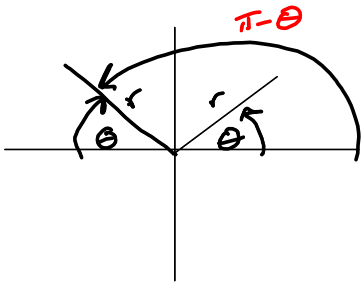
New p.

$r = 2\cos(\theta + \pi)$

$= 2\cos\theta\cos\pi - 2\sin\theta\sin\pi$

$= (2\cos\theta)(-1) - (2\sin\theta)(0)$

$= -2\cos\theta$ NOT SAME




Symmetric about $\theta = \frac{\pi}{2}$

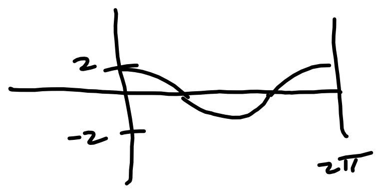
$r = 2\cos(\pi - \theta)$

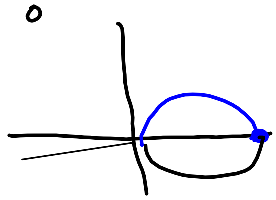
$= 2\cos(\pi)\cos(-\theta) - 2\sin(\pi)\sin(-\theta)$

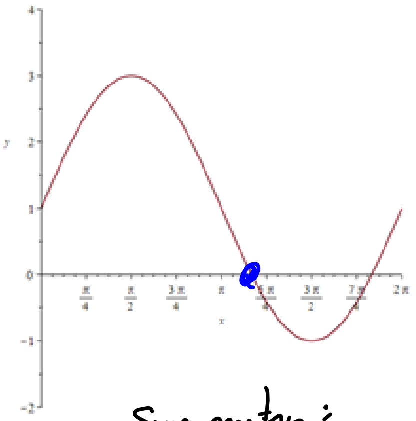
$= 2(-1)\cos\theta = -2\cos\theta$

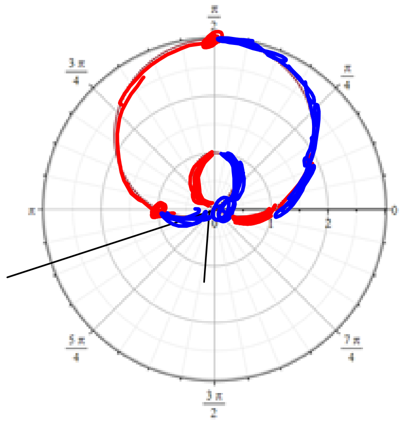
NOT SAME

$r = 2\cos\theta$ $\boxed{E}$ in Book





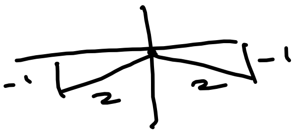


$2\sin\theta + 1 = 0$

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

$\theta =$




Symmetry:

$r = 1 + 2\sin\theta$

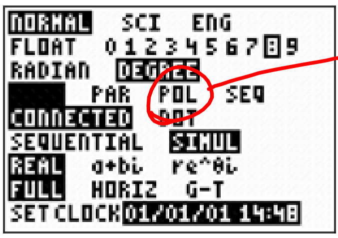
Test: $1 + 2\sin(\pi - \theta) = 1 + 2\underline{\sin\pi\cos(-\theta)} + 2\sin(-\theta)\cos\pi$

$= 1 + 2(-\sin\theta)(-1) = 1 + 2\sin\theta$

SAME!

$r = \sin(3\theta)$ 3-petal rose

$r = \cos(3\theta)$ 6-petal rose.




Zach points out that TI-84 & siblings have "POL" = polar mode for graphing polar eqns

Find where $r = \sin(3t)$ has horizontal tangents.

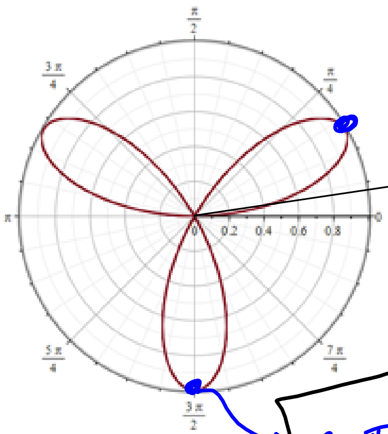
$y = \sin(3t)\sin(t)$

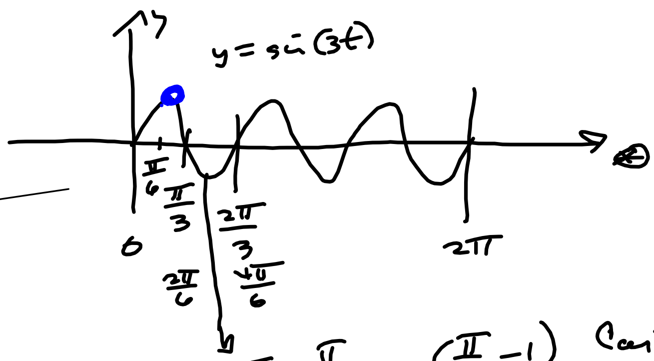
$x = \sin(3t)\cos t$ $\Longrightarrow$

$$\frac{dy}{d\theta} = \frac{3\cos(3t)\sin(t) + \sin(3t)\cos(t)}{3\cos(3t)\cos(t) - \sin(3t)\sin(t)} \quad \text{SET } 0$$

$\Longrightarrow$ $3\cos(3t)\sin(t) + \sin(3t)\cos(t) = 0$

is PAIN ful.




$\frac{3\pi}{6} = \frac{\pi}{2}$

$\left(\frac{\pi}{2}, -1\right)$ Cartesian.

$\left(-1, \frac{\pi}{2}\right) = \left(1, \frac{3\pi}{2}\right)$

Polar