Questions on 10.1 or 10.2?

Need to trim some of those "simulation" questions. I think Cengage lady tricked me into putting them on there....

10.2 #17

$x = 1+3t^2 \qquad 0 \le t \le 4$

$y = 7+2t^3$

$x' = 6t \longrightarrow (x')^2 = 36t^2$

$y' = 6t^2 \longrightarrow (y')^2 = 36t^4$

$$L = \int_0^4 \sqrt{36t^2+36t^4}\, dt = \int_\alpha^\beta ds = \int_\alpha^\beta \sqrt{(x')^2+(y')^2}\, dt$$

$$= \int_0^4 6t\sqrt{1+t^2}\, dt = 3\int_0^4 \sqrt{1+t^2}\,(2t\,dt) = 3\cdot \left.\frac{(1+t^2)^{3/2}}{\frac{3}{2}}\right]_0^4$$

$u = 1+t^2$

$du = 2t\,dt$

$$\sqrt{36t^2(1+t^2)} = \sqrt{36t^2}\,\sqrt{1+t^2}$$

$$= 6|t|\sqrt{1+t^2} = 6t\sqrt{1+t^2}$$

b/c $0 \le t \le 4$

$$2\left[(1+4^2)^{3/2} - (1)^{3/2}\right]$$

$$= 2\left[17^{3/2} - 1\right] = \left[(17^2 \cdot 17)^{\frac{1}{2}} - 1\right](2) = \boxed{2\left(17\sqrt{17} - 1\right)}$$

WebAssign Likes

$$17^{\frac{3}{2}} = 17^{\frac{2}{2}+\frac{1}{2}} = 17^1 \cdot 17^{\frac{1}{2}} = 17\sqrt{17}$$

10.2 #24

$x = a\cos^3\theta$, $y = a\sin^3\theta$ $\quad 0 \le \theta \le \frac{\pi}{2}$

Find surface area from revolving about x-axis

$A = 2\pi \int_\alpha^\beta y \underbrace{ds}_{\ge 0} = 2\pi \int_0^{\frac{\pi}{2}} a\sin^3\theta$

$x' = 3a\cos^2\theta(-\sin\theta)$

$= -3a\sin\theta\cos^2\theta \longrightarrow (x')^2 = 9a^2\sin^2\theta\cos^4\theta$

$y' = 3a\sin^2\theta\cos\theta \longrightarrow (y')^2 = 9a^2\sin^4\theta\cos^2\theta$

$ds = \sqrt{(x')^2 + (y')^2} = \sqrt{9a^2\sin^2\theta\cos^4\theta + 9a^2\sin^4\theta\cos^2\theta}$

$= \sqrt{9a^2\sin^2\theta\cos^2\theta(\cos^2\theta + \sin^2\theta)} = \sqrt{9a^2\sin^2\theta\cos^2\theta}$

$= 3|a\sin\theta\cos\theta| = 3|a|\cos\theta\sin\theta = \boxed{3a\cos\theta\sin\theta}$

WebAssign got sloppy about $|a|$ -vs- a

OR maybe I'm overlooking the fact that "y" in $2\pi\int_\alpha^\beta y\,ds$ needs to be ≥ 0.

Thanks, Zach.
Yes, I'm (now) recording.

$y = g(t)$ in definition of Parametric form, is assumed to be ≥ 0.

§8.2 says "$y \ge 0$" & so does §10.2, so $a > 0$, apparently.

Therefore, in the case where f is positive and has a continuous derivative, we define the **surface area** of the surface obtained by rotating the curve $y = f(x)$, $a \le x \le b$, about the x-axis as

4 $$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$$

So, $2\pi\int_0^{\frac{\pi}{2}} y\,ds = 2\pi\int_0^{\pi/2} a\sin^3\theta \cdot 3a\cos\theta\sin\theta\,d\theta$

$= 6\pi\int_0^{\frac{\pi}{2}} a^2\sin^4\theta\cdot\cos\theta\,d\theta = \frac{6a^2\pi\sin^5\theta}{5}\Big]_0^{\frac{\pi}{2}} = \frac{6a^2\pi}{5}$

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§10.2 #25 about y-axis $t\in[0,10]$

$\int te^t\,dt = uv - \int v\,du = \underline{te^t - e^t + C}$

$\left(\begin{array}{ll} u=t & dv=e^t\,dt \\ du=dt & v=e^t \end{array}\right.$ See §10.2 #25 $\Big)$ Scratch

$x=e^t-t \Longrightarrow x' = e^t-1 \Longrightarrow (x')^2 = e^{2t}-2e^t+1$

$y=4e^{\frac{t}{2}} \Longrightarrow y' = 2e^{\frac{t}{2}} \Longrightarrow (y')^2 = 4e^t$

$2\pi\int_0^{10} x\,ds = 2\pi\int_0^{10}(e^t-t)\sqrt{(e^t+1)^2}\,dt = 2\pi\int_0^{10}(e^t-t)(e^t+1)\,dt$

Scratch:

$(x')^2+(y')^2 = e^{2t}-2e^t+1+4e^t = e^{2t}+2e^t+1 = (e^t+1)^2$

$a^2+2a+1 = (a+1)^2$

$= 2\pi\int_0^{10}\left(e^{2t}+e^t-te^t-t\right)dt$

$= 2\pi\left[\frac{1}{2}e^{2t}+e^t-\underbrace{(te^t-e^t)}_{\text{see prev. work.}}-\frac{1}{2}t^2\right]_0^{10}$

$= 2\pi\left[\frac{1}{2}e^{20}-\frac{1}{2}e^0+e^{10}-e^0-\left(10e^{10}-e^{10}-[0e^0-e^0]\right)-\frac{1}{2}(10)^2\right]$

$= 2\pi\left[\frac{1}{2}e^{20}-\frac{1}{2}+e^{10}-1-10e^{10}+e^{10}-1-50\right]$

$= 2\pi\left[\frac{1}{2}e^{20}-\frac{3}{2}-51-8e^{10}\right] = \pi e^{20}-3\pi-102\pi-\cancel{8}e^{10}\pi$ 16

$= \pi e^{20}-\cancel{8}\pi e^{10}-105\pi$ 16

#26 §10.2

Curvature = $K = \kappa$ = Kappa = $\frac{1}{r}$, where r = radius of the osculating circle.

The straighter the curve, the larger the osculating circle's radius.

Big "r"
Not much curvature.

small "r"
Big Curvature

$$\kappa = \frac{y''}{\left(1+(y')^2\right)^{3/2}} = \text{curvature.}$$

$y = x^2$ Find curvature @ $(3, 9)$

$y' = 2x$

$y'' = 2$

$$\Rightarrow \kappa = \frac{2}{\left(1+(2x)^2\right)^{3/2}}$$

$$\Rightarrow \kappa(3) = \frac{2}{\left(1+6^2\right)^{3/2}} = \frac{2}{37^{3/2}} = \frac{2}{37\sqrt{37}}$$

Followup: where's curvature maximized?

$$\kappa'(x) = \frac{d}{dx}\left[2(1+4x^2)^{-3/2}\right]$$

$$= -3(1+4x^2)^{-5/2}(8x) = -\frac{24x}{\sqrt{(1+4x^2)^5}}$$

$+ \quad | \quad -$ $\quad \kappa'$

0

$\boxed{\text{Max @ } x=0}$

Curvature is $\boxed{\kappa(0) = 2}$