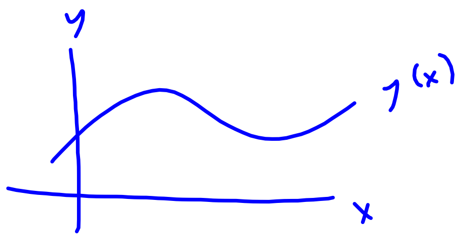
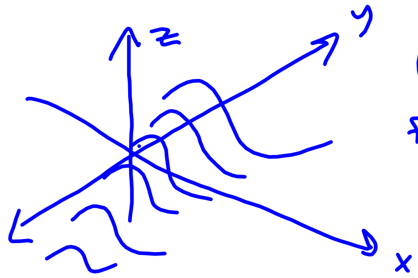
§ 9.5 Linear Ordinary Differential Equations (ODEs)

as opposed to Partial

for higher dimensions.





PDEs

Funcs are surfaces or Hypersurfaces.

$$y'(x) + P(x)\,y(x) = Q(x)$$

If the left-hand side is a derivative, we can integrate both sides & solve for $y(x)$.

$$\sin(x)\,y'(x) + \cos(x)\,y(x) = \sim$$

$$= \left(\sin(x)\,y(x)\right)' \text{ by product rule}$$

We're gonna MAKE the LHS the derivative of something.

$$\left(r(x)s(x)\right)' = y'(x) + P(x)\,y(x) = Q(x)$$

$$\longrightarrow \int \left(r(x)\,y(x)\right)' dx = \int Q(x)\,dx$$

& solve for $y(x)$.

If only we could find some function

$I(x) \ni$

$$I(x)\left[y'(x) + P(x)y(x)\right] = \left(I(x)y(x)\right)' = I(x)Q(x)$$

$$I(x)y'(x) + I(x)P(x)y(x) = I(x)Q(x)$$

$$= \left(I(x)y(x)\right)' = I'(x)y(x) + I(x)y'(x)$$

$$= I(x)y'(x) + I(x)P(x)y(x) = I(x)Q(x)$$

$$\Longrightarrow I'(x)y(x) = I(x)P(x)y(x)$$

$$\Longrightarrow I'(x) = I(x)P(x)$$

$$\Longrightarrow \frac{I'(x)}{I(x)} = P(x)$$

$$\Longrightarrow \int \frac{I'(x)}{I(x)}\,dx = \int P(x)\,dx$$

$$\Longrightarrow \ln|I(x)| = \int P(x)\,dx + \hat{c} \qquad a^{b+c} = a^b a^c$$

$$\Longrightarrow e^{\ln|I(x)|} = |I(x)| = e^{\int P(x)\,dx + \hat{c}} = e^{\int P(x)\,dx}e^{\hat{c}}$$

$$= Ce^{\int P(x)\,dx}$$

$$I(x) = \pm Ce^{\int P(x)\,dx}$$

SET $I(x) = Ce^{\int P(x)\,dx} = e^{\int P(x)\,dx}$ if $c = 1$ & $I(x)$ does what I want.

Your integrating factor;

$$I(x) = e^{\int P(x)\,dx}$$

$y(x) + x\,y'(x)$ would've been better than $\sin(x)\,y(x) + \cos(x)\,y'(x)$

$$y'(x) + 1y(x) = \cos(x)$$

$p(x) = 1$, $Q(x) = \cos(x)$

$$\Rightarrow I(x) = e^{\int 1\,dx} = e^{x+c} = e^x \text{ if } c = 0$$

$$\Rightarrow e^x y'(x) + e^x y(x) = e^x\cos(x)$$

$$\left(e^x y(x)\right)' = e^x \cos(x)$$

$$\Rightarrow e^x y(x) = \int e^x \cos(x)\,dx = \tfrac{1}{2}e^x\left[\sin(x) + \cos(x)\right] + C$$

$$\Rightarrow y(x) = \frac{1}{e^x}\cdot\left(\tfrac{1}{2}e^x\left[\sin(x) + \cos(x)\right] + C\right)$$

$$= \tfrac{1}{2}\left(\sin(x) + \cos(x)\right) + Ce^{-x}$$

In general,

$$y(x) = \frac{1}{I(x)}\left[\int I(x)Q(x)\,dx + C\right]$$

Want $I(x)$ $\ni$ $I(x)y'(x) + I(x)P(x)y(x)$

$$= \left(I(x)y(x)\right)'$$

$f(x)$

function of x

It's all English!

It all good.

15–20 Solve the initial-value problem.

15. $x^2y' + 2xy = \ln x, \quad y(1) = 2$

Already written as a derivative of x^2y! Take a step back! Let the patterns emerge.

$$y' + \frac{2}{x}y = \frac{1}{x^2}\ln(x)$$

$$I(x) : e^{\int \frac{2}{x}dx} = e^{2\ln|x| + \hat{C}} \Rightarrow$$

$$\cancel{I(x) = \ \_\_\_}$$

$$= e^{2\ln|x| + \hat{C}} = ce^{\ln|x^2|} = c|x^2| = cx^2 = x^2, \text{ if } c = 1 \text{ which is fine.}$$

$$\text{Let } I(x) = x^2$$

Unnecessary, but good check:

$$x^2y' + 2xy = \frac{d}{dx}[x^2y] = x^2\left(\frac{1}{x^2}\ln(x)\right) = \ln(x)$$

$$\Rightarrow \quad x^2y = \int \ln(x)\,dx = x\ln(x) - x + C$$

$u = \ln(x)$ $\quad dv = dx$

$du = \frac{1}{x}dx$ $\quad v = x$

$$uv - \int v\,du = x\ln(x) - \int x \cdot \frac{1}{x}dx$$

$$= x\ln(x) - x + C$$

$$\Rightarrow \quad y = \frac{x\ln(x) - x + C}{x^2} = \frac{\ln(x)}{x} - \frac{1}{x} + \frac{C}{x^2}$$