$y' = ky$ Exponential

$y' = ky(M-y)$ OR $ky(1 - \frac{y}{M})$ M = carrying capacity

$m \frac{d^2y}{dt^2} = -ky$

DEPlot in Maple

Euler's Method Approximate values for the solution of the initial-value problem $y' = F(x, y)$, $y(x_0) = y_0$, with step size h, at $x_n = x_{n-1} + h$, are

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}) \qquad n = 1, 2, 3, \ldots$$

§9.1 Differential Equations

The most common is the Exponential

$y' = ky$ growth rate is proportional to population.

$\Longrightarrow k = \frac{y'}{y}$

$$kt = \int k\,dt = \int \frac{y'\,dt}{y} = \ln|y| + \hat{C}$$

$\ln|y| = kt + C$ Pop? $y \geq 0$ $(C = -\hat{C})$

$$y = e^{\ln(y)} = e^{kt+C} = e^{kt}e^{C} = e^{C}e^{kt} = \tilde{C}e^{kt} \quad (\tilde{C} = e^{C})$$

$y = \tilde{C}e^{kt}$ for some constants $\tilde{C}, k$.

k = relative growth rate

$\tilde{C} = y(0) = y_0$ = Initial population.

uninhibited growth. Not very realistic.

Pop. Model:

Want $y' = 0$ when $y = 0$.

Want y' to taper off when $y \longrightarrow$ Big.

Let M = carrying capacity of the ecosystem.

$$y' = ky(M-y) > 0 \text{ until } y > M.$$

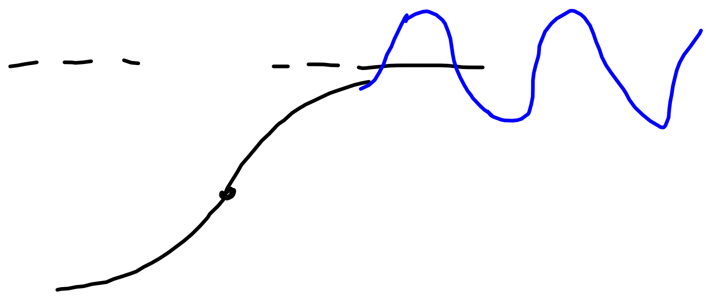
$$= Mky\left(1-\frac{y}{M}\right) = \hat{k}\, y\left(1-\frac{y}{M}\right) \text{ is Book model.}$$

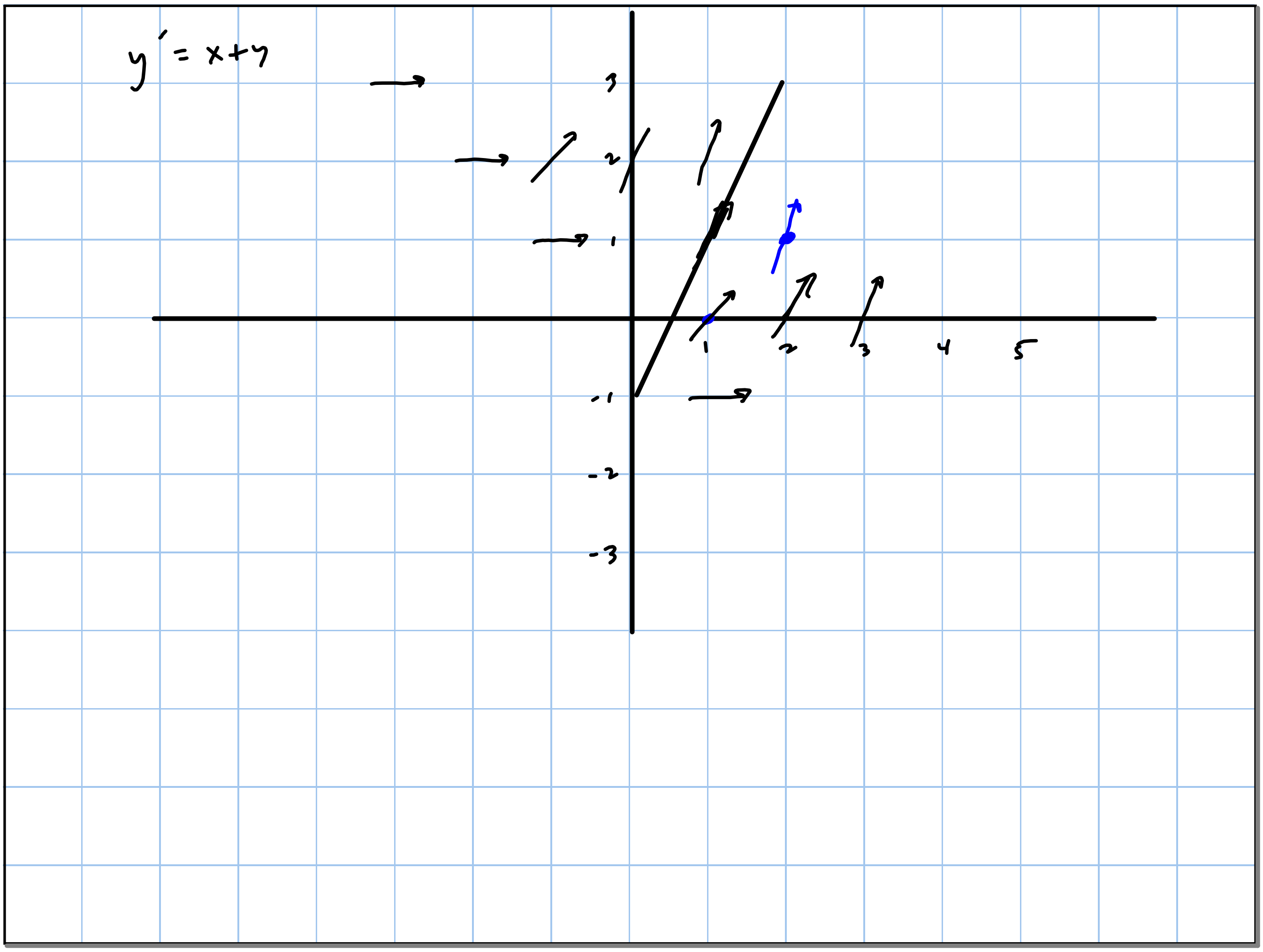
$$(\hat{k} = MK)$$

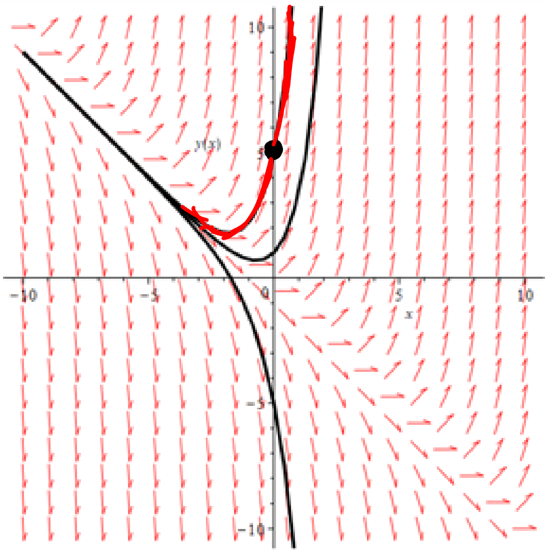
with(DEtools);

$y' = x+y$

$\nearrow \; \rightarrow \; \searrow$




y' = x+y
3
2
1
-1
-2
-3
1
2
3
4
5



$y = -x-1$

$y' = x + (-x-1) = -1$

has equilibrium solution along the line $y = -1-x$ w/ slope $y' = -1$

DEplot(Diff(y(x), x) = x + y(x), y(x), x = -10 .. 10, y = -10 .. 10, [y(0) = 1, y(0) = 5, y(0) = -5], linecolor = black)

§9.1 Very general ideas.

No general technique.

$y=0$ & you're done

I invented the "Marlin Perkins Term."

to guarantee the existence of positive solutions!

Jim

Lotka-Volterra Model

Euler's Method, by hand, is PAINFUL. But you can do a lot with DEtools in Maple.

Euler's Method Approximate values for the solution of the initial-value problem $y' = F(x, y)$, $y(x_0) = y_0$, with step size h, at $x_n = x_{n-1} + h$, are

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}) \qquad n = 1, 2, 3, \ldots$$

Start with $(x_0, y_0) = (x_0, y_0(x_0))$ as 1^{st} point

use $y' = F(x_0, y_0)$ to get a tangent to y.

Draw a little tangent segment / Use tangent line over the interval $[x_0, x_0 + \text{STEP-SIZE}] = [x_0, x_0+h]$ to

$(x_1, y_1) = (x_0+h, y_0 + h\, m_{tan}) = (x_0+h, y_0 + hF(x_0, y_0))$

Rinse & Repeat.

$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$

You can build a table of values.

Let's do Euler's for $y' = x+y$ in Excel.

$y_1 =$

F4- Absolute Reference

$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}) = y_{n-1} + h(x_{n-1} + y_{n-1})$

Spring Model Hooke's Law
weight hanging from a spring.

$$m \frac{d^2x}{dt^2} = F = -Kx(t)$$

Ohm's Law & Kirchoff's Law

Schedule says 9.1, 9.2 today
9.2 tomorrow.

It's always nice to have a simple

$y' = f(x)$

Just integrate to the solution

$y' = x^3 \longrightarrow$

$y = \frac{x^4}{4} + C \qquad (\forall C \in \mathbb{R})$