Questions?

Consumer Surplus

Probability Distribution for a random variables.

2 Dice

$6 \times 6 = 36$

x	n	p
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	1/6
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

$1 = \sum p_k$

Expected Value

$$= \sum p_k x_k = \frac{1}{36}\cdot 2 + \frac{2}{36}\cdot 3 + \cdots + \frac{2}{36}\cdot 11 + \frac{1}{36}\cdot 12$$

$$= 7 = E(x)$$

We'll be dealing with

Continuous Random Variables.

$$\int p(x)\, x \, dx = \text{Expected Value}$$

Big Part of the exercises is

Show $\int_a^b p(x)\,dx = 1$

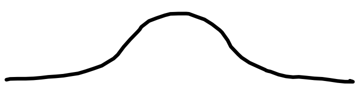
$$P(a \le x \le b) = \int_a^b p(x)\,dx$$

Bell Curve

Normal Distribution

To make it "1."

$$c\int_{-\infty}^{\infty} e^{-x^2}\,dx$$



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

Standard Normal Distribution

σ = Standard Deviation

$$\sigma^2 = \sum_{k=1}^{n} (\bar{x} - x_k)^2 = \text{variance}$$

Discrete (Not Continuous)

$$\bar{x} = \frac{\sum_{k=1}^{n} x_k}{n}$$

Frequency Data

$$\sum_{k=1}^{n} f_k (\bar{x} - x_k)^2 \qquad \bar{x} = \frac{\sum_{k=1}^{n} f_k x_k}{\sum_{k=1}^{n} f_k} \xrightarrow{n \to \infty} \frac{\int f(x)\, x\, dx}{\int f(x)\, dx}$$

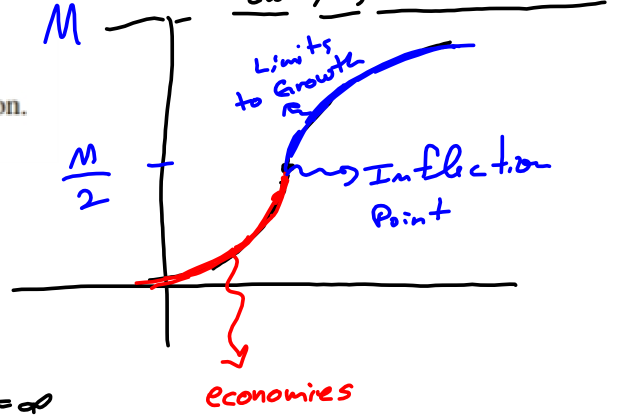
4. The density function

$$f(x) = \frac{e^{3-x}}{(1 + e^{3-x})^2}$$

is an example of a *logistic distribution*.
(a) Verify that f is a probability density function.
(b) Find $P(3 \leqslant X \leqslant 4)$.

Intuitive starting point for population modeling.
Never witnessed in Nature (not exactly).

Carrying Capacity



(a) $$\int_{-\infty}^{\infty} \frac{e^{3-x}}{(1+e^{3-x})^2}\, dx = -\int_{x=-\infty}^{x=\infty} \frac{du}{1+u^2}$$

$u = e^{3-x}$

$du = -e^{3-x}\, dx$

$$= -\int_{x=-\infty}^{\infty = x} \frac{du}{1+u^2} = -\arctan(u)\Big]_{x=-\infty}^{x=\infty}$$

$$= \lim_{t\to\infty} \left(-\arctan(3-x)\right)\Big|_{-t}^{t} = \lim_{t\to\infty} \left(-\arctan(3-t)\right) - \left(-\arctan(3-(-t))\right)$$

Dude! $e^{3-t} = u$!

$$= \left(-\left(-\frac{\pi}{2}\right)\right) - \left(-\frac{\pi}{2}\right)\Big) = \pi$$

$$3-t \xrightarrow{t\to\infty} -\infty$$

$$3+t \xrightarrow{t\to+\infty} \infty$$

∞

$$\int_{-\infty}^{+\infty} \frac{e^{3-x}}{(1+e^{3-x})^2}\,dx$$

$u = e^{3-x}$

$du = -e^{3-x}\,dx$

$$= -\int_{-t}^{t} \frac{du}{(1+u)^2} = -\int_{-t=x}^{t=x} \frac{dv}{v^2} = -\frac{v^{-1}}{-1}\Bigg]_{x=-t}^{x=t}$$

$v = 1+u$

$dv = du$

$$\frac{1}{1+u}\Bigg]_{x=-t}^{x=t} = \frac{1}{1+e^{3-x}}\Bigg]_{-t=x}^{t=x} \text{“}=\text{”} \left[\frac{1}{1+e^{3-t}} - \frac{1}{1+e^{3+t}}\right]$$

$$\xrightarrow{t\to\infty} \frac{1}{1+0} - \frac{1}{1+\text{“}\infty\text{”}} = 1$$

Write much.
Think little, mahstah.
Correct, grahsshoppah.

(b) $\int_3^4 \frac{e^{3-x}}{(1+e^{3-x})^2}\,dx = P(3 \le X \le 4)$

$$\left.\frac{1}{1+e^{3-x}}\right]_3^4 = \frac{1}{1+e^{-1}} - \frac{1}{1+e^0} = \frac{1}{1+\frac{1}{e}} - \frac{1}{2}$$

$$= \frac{1}{\frac{e+1}{e}} - \frac{1}{2} = \frac{e}{e+1} - \frac{1}{2} = \frac{2e-(e+1)}{2(e+1)}$$

$$= \frac{e-1}{2e+2}$$

20. The standard deviation for a random variable with probability density function f and mean μ is defined by

$$\sigma = \left[\int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx \right]^{1/2}$$

I used $(\bar{x} - x_i)^2$

They're doing $(x - \bar{x})^2 = (x - \mu)^2$