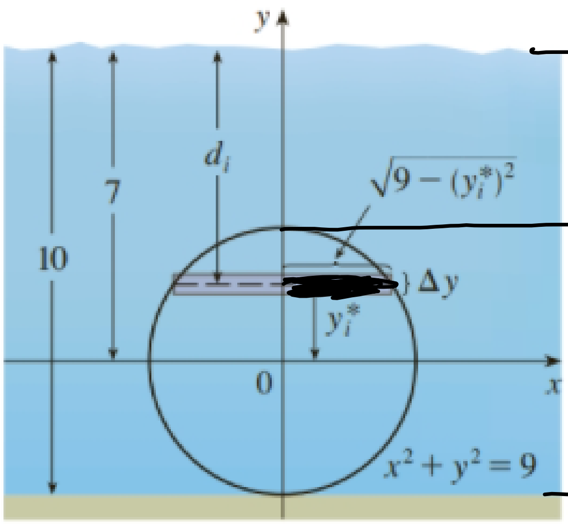


$0 = y = d$

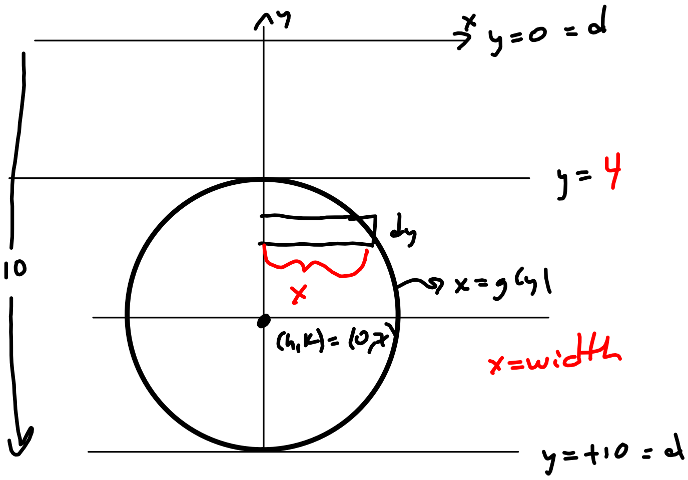
$2\int_4^{10}$

$y = 4$

$y = 10$

$F = mg = \rho g A d$ $\quad \rho = 62.5 \text{ lb/ft}^3$

$A = \text{area in ft}^2 = \text{width} \cdot dy$



$$2\int_4^{10} 62.5\, \overset{\text{depth}=d}{y} \underbrace{\sqrt{9-(y-7)^2}}_{w} \cdot \underbrace{dy}_{h} = A$$

$(h,k) = (0,7),\ r = 3$

$x^2 + (y-7)^2 = 3^2$

$x = \pm\sqrt{9-(y-7)^2}$

take the positive.

$$2 \cdot 62.5 \int_4^{10} y\sqrt{9-(y-7)^2}\, dy$$

Let $u = y-7 \Rightarrow y = u+7$

$du = dy$

$y = 4 = u+7 \Rightarrow u = -3$

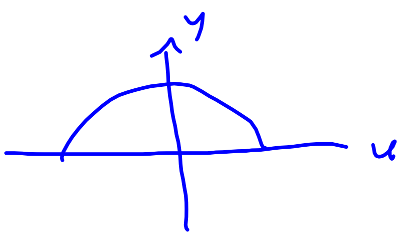
$y = 10 = u+7 \Rightarrow u = 3$

$$125\int_{-3}^{3} (u+7)\sqrt{9-u^2}\, du$$

$$= \frac{-125}{2}\int_{-3}^{3} -2u\underbrace{\sqrt{9-u^2}}\, du + 7 \cdot 125\int_{-3}^{3}\sqrt{9-u^2}\, du$$

$v = 9-u^2$

$dv = -2v\, dv$



$$7 \cdot 125 \cdot \frac{\pi \cdot 3^2}{2}$$

Moment:

Recall Average of $x_1, x_2, x_3, \ldots, x_n$

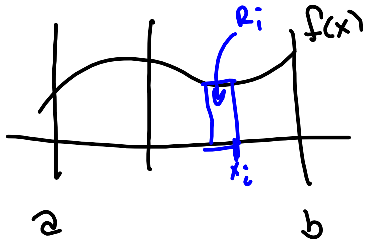
$$\Rightarrow \bar{x} = \frac{\sum x_k}{n}$$

Average from frequency data

x	f
x_1	f_1
⋮	⋮
x_n	f_n

$$\bar{x} = \frac{\sum f_k x_k}{\sum f_k}$$

Now we're talking about the Moment about the x- & y-axes.



M_y = moment about the y-axis is the "mass times position" added up

$$M_y \approx \sum M_{y_i} = \sum_{i=1}^{n} \overset{\text{mass}}{m_i}\, x_i \xrightarrow{n\to\infty} \int_a^b x\, m(x)\, dx$$

$$\bar{x} = \frac{M_y}{\text{mass}} = \frac{\int_a^b x\, m(x)}{\int_a^b m(x)} = \boxed{\frac{\rho \int_a^b x\, f(x)\,dx}{\rho \int_a^b f(x)\,dx}} = \frac{\int_a^b x\, f(x)\,dx}{\int_a^b f(x)\,dx}$$

GOOD

$m(x)$ = density times area $= \rho\, f(x)\,dx$

$$\bar{y} = \frac{M_x}{\text{Mass}} \approx \frac{\sum \rho\, \frac{1}{2} f(x_i) \cdot f(x)\,\Delta x}{\text{Mass}} = \rho \sum \tfrac{1}{2} f(x)^2\,dx$$

$\frac{1}{2} f(x_i)$ is the mid-height for the i^{th} rectangle.

Moment Problems: Drudgery – Maple!