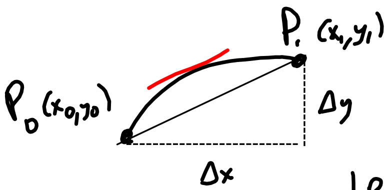
§8.1 Arc Length

WP#2 will be over Test 3 segment. Don't listen to WebAssign.

Arc Length



We approximate arc length w/ length of the line segment $\overline{P_0P_1}$

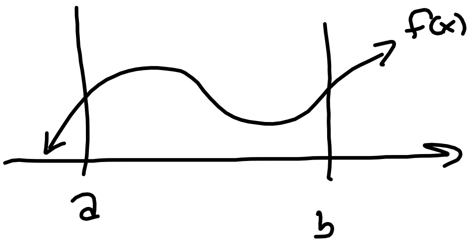
$$|P_0P_1| = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2} \approx \text{Arc length}$$

$$|\overline{P_0P_1}| = \sqrt{(\Delta x)^2+(\Delta y)^2}\cdot\frac{\Delta x}{\Delta x} = \sqrt{(\Delta x)^2+(\Delta y)^2}\,\frac{\Delta x}{\sqrt{(\Delta x)^2}}$$

$$= \sqrt{\left(\frac{\Delta x}{\Delta x}\right)^2+\left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x = \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x \xrightarrow{\Delta x\to 0}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = ds = \text{differential of arc length}$$

"=" increment of arc length.



L = Arc length over $[a,b]$

$$= \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

$$= \int_a^b ds$$

Our 1st Application is §8.2:
Surface Area of a solid of revolution

ALSO, line integrals, like force acting along a curve. Work done by rocket engine.

$\int f(t)\,ds$ Work done by fields on a particle following a path.

ds
$= ds(t)$, i.e.,
as a function of t.

$f(x) = y = 1+6x^{\frac{3}{2}}$, $x \in [0,1]$ Find Arc Length

$f'(x) = \frac{3}{2}(6)x^{\frac{1}{2}} = 9x^{\frac{1}{2}}$

$\Rightarrow (f'(x))^2 = 81x \longrightarrow$

$L = \int_0^1 \sqrt{1+81x}\,dx = \frac{1}{81}\int_0^1 (81x+1)^{\frac{1}{2}}(81dx)$

DOH!

$= \frac{1}{81}\left[\frac{2}{3}(81x+1)^{\frac{3}{2}}\right]_0^1 = \frac{2}{243}\left[(81+1)^{\frac{3}{2}} - (0+1)^{\frac{3}{2}}\right]$

$\frac{2}{243}\cdot 81 \neq \boxed{\frac{2}{3}}$

Newp

$= \frac{2}{243}\left((82)^{\frac{3}{2}} - 1\right)$

$= \frac{164}{243}\sqrt{82} - \frac{2}{243}$

$2 \,\underline{)\,82}$
41

$82^{\frac{3}{2}} = \sqrt{82^2 \cdot 82} = 82\sqrt{82}$

Ryan's TI-34 Multi-View Sucks.

Don't use as a crutch

$y = \frac{x^3}{3} + \frac{1}{4x}$ $\quad 1 \le x \le 2$

$y' = x^2 - \frac{1}{4x^2} \Longrightarrow (y')^2 = x^4 - 2(x^2)\left(\frac{1}{4x^2}\right) + \frac{1}{16x^4} = x^4 - \frac{1}{2} + \frac{1}{16x^4}$

$\frac{d}{dx}(4x)^{-1} = -(4x)^{-2}(4) = \frac{-4}{4^2x^2} = -\frac{1}{4x^2}$

$\Longrightarrow L = \int_1^2 \sqrt{1+(y')^2}\,dx = \int_1^2 \left(x^4 + \frac{1}{2} + \frac{1}{16x^4}\right)^{\frac{1}{2}} dx$

$= \int_1^2 \left(\left(x^2 + \frac{1}{(4x)^2}\right)^2\right)^{\frac{1}{2}} dx = \int_1^2 \left(x^2 + \frac{1}{16}x^{-2}\right) dx$

$= \left[\frac{x^3}{3} - \frac{1}{16}x^{-1}\right]_1^2 = \left(\frac{2^3}{3} - \frac{1}{16}(2^{-1})\right) - \left(\frac{1}{3} - \frac{1}{16}\right)$

$= \frac{8-1}{3} - \frac{1}{32} + \frac{1}{16}\cdot\frac{2}{2} = \frac{7}{3} + \frac{1}{32} = \frac{224+3}{96} = \frac{227}{96}\,?$

$x^4 + \frac{1}{2} + \frac{1}{16x^4} = u^2 + \frac{1}{2} + \frac{1}{(4u)^2}$

$u^2 + \frac{1}{2}$ Bleah! n1 failed us!

Recognizing a Perfect Square Trinomial

$a^2 \pm 2ab + b^2$

$x^4 = a^2 = (x^2)^2$

$\frac{1}{16x^4} = \frac{1}{(4x^2)^2} = \left(\frac{1}{4x^2}\right)^2 = b^2$

& $2ab = 2(x^2)\left(\frac{1}{4x^2}\right) = \frac{2}{4} = \frac{1}{2}$ ✓

$2 \,\lfloor 96$
$2 \,\lfloor 48$
$2 \,\lfloor 24$
$2 \,\lfloor 12$
$2 \,\lfloor 6$
3

$\lfloor 227$

https://harryzaims.com/202/videos/chapter-08/8-2-surface-area-of-solids-of-revolution/8-2-theory.mp4

§ 8.2 Surface Area of a Surface of Revolution

See theory video

Rotating about the x-axis:

$$A = 2\pi\int_a^b \underbrace{y}_{\text{radius}}\, ds \quad\to\quad 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx \quad y=f(x)$$

$$\to\quad 2\pi\int_a^b y\sqrt{1+(g'(y))^2}\,dy \quad \text{if } x=g(y)$$

Rotating about the y-axis

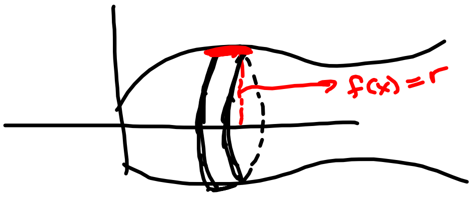
$$A = 2\pi\int_a^b \underbrace{x}_{\text{radius}}\, ds \quad\to\quad 2\pi\int_a^b g(y)\sqrt{1+(g'(y))^2}\,dy \quad x=g(y)$$

$$\to\quad 2\pi\int_a^b x\sqrt{1+(f'(x))^2}\,dx \quad y=f(x)$$

See Video:

https://harryzaims.com/202/videos/chapter-08/8-2-surface-area-of-solids-of-revolution/8-2-theory.mp4

General Idea



Finding Area of Frustum of a Cone.

a) 11:08 in video, we see

Area of one of those frusta is

$$2\pi r \ell = 2\pi f(x)\, ds$$

$$\therefore\quad 2\pi\int_a^b y\, ds = 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$