Comparisons....

49–54 Use the Comparison Theorem to determine whether the integral is convergent or divergent.

49. $\int_0^\infty \frac{x}{x^3+1}\,dx$ **50.** $\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx$

51. $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$ **52.** $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$

53. $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx$ **54.** $\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$

$\frac{1}{x^2}$

$\frac{x}{x^3+1} < \frac{x}{x^3} = \frac{1}{x^2}$ converges

$\geq \frac{1}{\sqrt{x}}$ diverges

Think $\frac{x}{\sqrt{x^4}} = \frac{x}{x^2} = \frac{1}{x}$

Diverges

$\frac{x+1}{\sqrt{x^4-x}} > \frac{x}{\sqrt{x^4-x}} > \frac{x}{\sqrt{x^4}} = \frac{x}{x^2}$

$\frac{1}{4-1} > \frac{1}{4}$ $= \frac{1}{x}$

Diverges

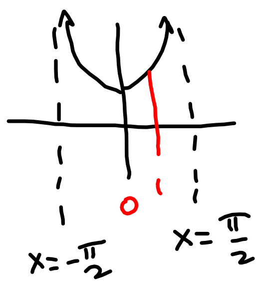
52 $\int_0^\infty \frac{\arctan(x)}{2+e^x}$

$\frac{\arctan(x)}{2+e^x} \leq \frac{\frac{\pi}{2}}{2+e^x} < \frac{\frac{\pi}{2}}{e^x} = \frac{\pi}{2}\left(e^{-x}\right)$ converges

Growth rates $\ln(x) < x^{\text{pos.}} < b^x$

log < power < exponential

$\frac{1}{\text{log}} > \frac{1}{\text{power}} > \frac{1}{\text{exponential}}$



on $[0,1]$, $\sec(x)$ is bdd by $\sec(1)$ on $[0,1]$

$\int_0^1 \frac{\sec^2(x)}{x\sqrt{x}}$

Bdd below by $y=1$, so

$\frac{\sec^2(x)}{x\sqrt{x}} \geq \frac{1}{x\sqrt{x}}$

$= \frac{1}{x^{3/2}}$ Diverges.

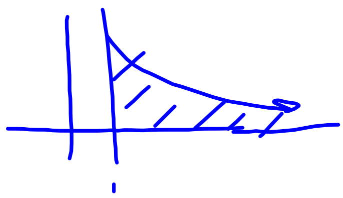
Type I $\int_1^\infty \frac{dx}{x^{3/2}}$ CONVERGES

$\int_0^3 \frac{dx}{x^{3/2}}$ diverges

Type II $\int_0^1 \frac{1}{x^{1.5}}$

Function Blows up @ $x=0$ which makes it improper.

Flunks p-test.



$$\int_0^{\pi} \frac{\sin^2(x)}{\sqrt{x}}$$

Diverges: Find something smaller that diverges
Converges: " " bigger " converges

$$\frac{\sin^2(x)}{\sqrt{x}} \le \frac{1}{\sqrt{x}} \quad \& \quad \int_0^{\pi} \frac{dx}{\sqrt{x}} \text{ converges by p-test}$$

They're avoiding $\int_3^{\infty} \frac{dx}{x^2-x-2}$ converges, but it's hard to compare it directly to $\frac{1}{x^2}$

$$\frac{1}{x^2+x+2} \le \frac{1}{x^2}$$

$$\frac{1}{x^2-x-2} \overset{?}{\le}$$

Notation Abuse! (Need limits!)

$$\int_3^{\infty} \frac{A\,dx}{x+1} + \int_3^{\infty} \frac{B\,dx}{x-2}$$

$$= \Big[A \ln|x+1|\Big]_3^{\text{"}\infty\text{"}} + \Big[B \ln|x-2|\Big]_3^{\text{"}\infty\text{"}}$$

$$\pm\infty \qquad \mp\infty$$

Very Closely related to proving

$$\lim_{x\to 5}(x^2-x-2)=18$$

Want $|x^2-x-2-18|<\varepsilon$

$\Longrightarrow |x^2-x-20| = |x-5||x+4| < \varepsilon$

whenever $|x-5|<\delta$

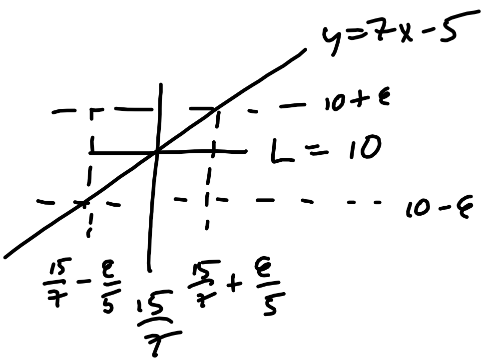
$|x+1||x-5| < |x+1|\delta < 7\delta \le 7\cdot\frac{\varepsilon}{7}=\varepsilon$

Assume $\delta \le 1 \Longrightarrow$

$4<x<6$

$5 < x+1 < 7$

So make $\delta = \min\left\{1, \frac{\varepsilon}{7}\right\}$



Let $\varepsilon>0$

$7x-5=10$

$7x=15$

$x=\frac{15}{7}$