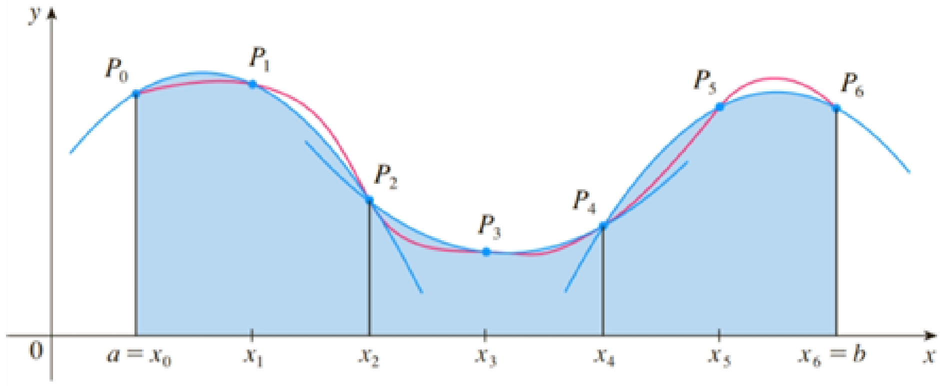
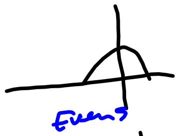
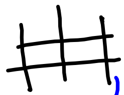
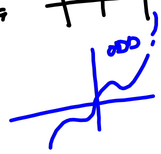
Simpson's Rule

Use parabolas to approximate f(x) between 3 points

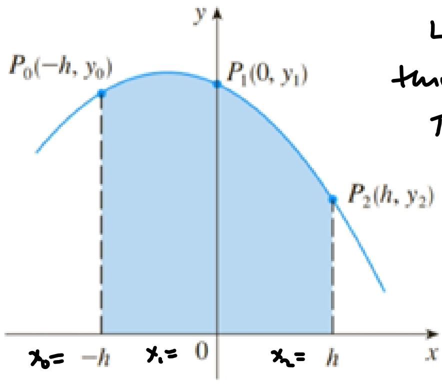








Derivation:



Assume middle point is $(0, f(0))$

Let $Ax^2+Bx+C = y$ be the parabola

thru P_0, P_1, P_2.

Even ODD

Then Area $= \int_{-h}^{h} (Ax^2+Bx+C)\,dx =$

$2\int_0^h (Ax^2+C)\,dx = 2\left[A\frac{x^3}{3} + Cx\right]_0^h$. why?

$= 2\left[A\frac{h^3}{3} + Ch\right] = \frac{2Ah^3}{3} + 2Ch$

$= \frac{h}{3}\left[2Ah^2 + 6C\right]$

Also $(-h, y_0)$, $(0, y_1)$, & (h, y_2) are on the parabola, so

$$\left.\begin{aligned} y_0 &= A(-h)^2 + B(-h) + C = Ah^2 - Bh + C \\ y_1 &= C \\ y_2 &= Ah^2 + Bh + C \end{aligned}\right\} \Rightarrow$$

$$y_0 + 4y_1 + y_2 = 2Ah^2 + 6C$$

$\therefore$ Area $= \frac{h}{3}(2Ah^2 + 6C) = \boxed{\frac{h}{3}\left[y_0 + 4y_1 + y_2\right] = \text{Area!}}$

Add up the areas under ALL the parabolas:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\Big[y_0+4y_1+y_2 + y_2+4y_3+y_4 + y_4+4y_5+y_6+\cdots$$
$$+ y_{n-2}+y_{n-2}+4y_{n-1}+y_n\Big]$$

$$= \frac{\Delta x}{3}\Big[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots$$
$$+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\Big]$$

$$= \frac{b-a}{3n}\left[f(x_0)+\sum_{k=1}^{\frac{n-2}{2}\,?}\big(4f(x_{2k-1})+2f(x_{2k})\big)+f(x_n)\right]$$

Nice Try

Took a stab.

MAPLE IMPLEMENTATION

Tools > Tutors > Calculus - single variable > Approximate Integration

I started w/ MACSYMA

If you're doing these w/ calculator, DO NO ROUNDING until the very end.

Error Propagation is a BEAST!

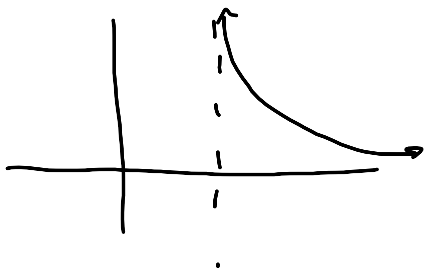
§ 7.8

$\int_1^\infty \frac{1}{x^p}\,dx$ is convergent if $p > 1$ and divergent if $p \leqslant 1$. TYPE I

TYPE II: Finite interval; FUNCTION BLOWS UP.

EXAMPLE 5 Find $\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$.

Like $\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$ @ $x=0$



$$\int_t^5 \frac{1}{\sqrt{x-2}}\,dx = \int_t^5 (x-2)^{-\frac{1}{2}}dx$$

$$= \int_{x=t}^{x=5} u^{-\frac{1}{2}}du = \left[2(x-2)^{\frac{1}{2}}\right]_t^5 \text{ where}$$

$u = x-2$
$du = dx$

$$= 2(5-2)^{\frac{1}{2}} - 2(t-2)^{\frac{1}{2}} = 2\sqrt{3} - 2\sqrt{t-2}$$

$$\xrightarrow{t\to 2} \boxed{2\sqrt{3}}$$

You REALLY want to get good @ TYPE I's

$\int_1^\infty \frac{dx}{x^2}$ converges $\underbrace{\text{if and only if}}_{\Longleftrightarrow \text{ iff}}$ $\sum_{n=1}^\infty \frac{1}{n^2}$ converges

In later chapters

$\int_1^\infty f(x)\,dx$ converges iff $\sum_{n=1}^\infty f(n)$ converges

INTEGRAL TEST!

Section 7.3

6. 0/1 points

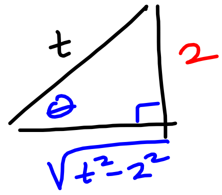
Evaluate the integral.

$$\int_{2\sqrt{2}}^{4} \frac{1}{t^3\sqrt{t^2-4}}\,dt$$

$$-\frac{1}{32}+\frac{\sqrt{3}}{64}+\frac{\pi}{192}$$

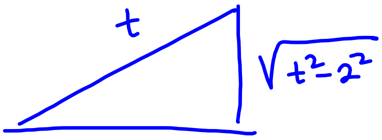
$\sqrt{t^2-4} = \sqrt{t^2-2^2}$

t = hypot.



$\frac{2}{t} = \sin\theta$

$\frac{t}{2} = \csc\theta$



$\frac{t}{2} = \sec\theta$

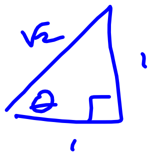
$t = 2\sec\theta$

$dt = 2\sec\theta\tan\theta\,d\theta$

$t = 2\sqrt{2} = 2\sec\theta$

$\sec\theta = \sqrt{2}$

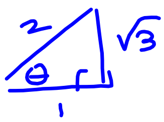
$\theta = \frac{\pi}{4}$



$t = 4 = 2\sec\theta$

$\sec\theta = 2$

$\theta = \frac{\pi}{3}$



6. 0/1 points

Evaluate the integral.

$$\int_{2\sqrt{2}}^{4} \frac{1}{t^3\sqrt{t^2-4}}\,dt$$

$$-\frac{1}{32}+\frac{\sqrt{3}}{64}+\frac{\pi}{192}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\sec\theta\tan\theta\,d\theta}{2^3\sec^3\theta\cdot 2\tan\theta}$$

$$= \frac{1}{8}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{d\theta}{\sec^2\theta}$$

$$= \frac{1}{8}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2\theta\,d\theta = \frac{1}{8}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\cos(2\theta)}{2}\,d\theta$$

$$= \frac{1}{8}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\frac{1}{2}+\frac{1}{2}\cos(2\theta)\right)d\theta$$

$$= \frac{1}{8}\int_{\frac{\pi}{4}}^{\pi} \frac{1}{2}\,d\theta + \frac{1}{8}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2}\cos(2\theta)\,d\theta$$

$u = 2\theta$

$du = 2\,d\theta$

$\frac{du}{2} = d\theta$

$$\sqrt{t^2-2^2}$$
$$= \sqrt{2^2\sec^2\theta - 2^2}$$
$$= 2\sqrt{\sec^2\theta - 1}$$
$$= 2\sqrt{\tan^2\theta}$$
$$= 2|\tan\theta|$$
$$= 2\tan\theta$$

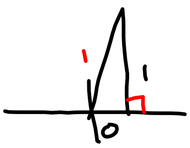
$$= \frac{1}{16}\left[\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} + \frac{1}{8}\int_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{3}} \frac{1}{2}\cos(u)\,\frac{du}{2}$$

$$= \frac{1}{16}\left[\frac{\pi}{3}\cdot\frac{4}{4} - \frac{\pi}{4}\cdot\frac{3}{3}\right] + \frac{1}{32}\left[\sin(u)\right]_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{3}}$$

$$\begin{array}{r}12\\10\\\hline 120\end{array} + \begin{array}{r}12\\6\\\hline 72\end{array}$$

$$= \frac{1}{16}\left[\frac{\pi}{12}\right] - \frac{1}{32}\left[\sin(2\theta)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} =$$

$$\frac{\pi}{192} - \frac{1}{32}\left[\sin\left(\frac{2\pi}{3}\right) - \sin\left(\frac{2\pi}{4}\right)\right] = \frac{\pi}{192} - \frac{1}{32}\left[\frac{\sqrt{3}}{2} - 1\right]$$



$$= \frac{\pi}{192} - \left(\frac{\sqrt{3}}{64} - \frac{1}{32}\right) = \boxed{\frac{\pi}{192} - \frac{\sqrt{3}}{64} + \frac{1}{32}}$$

Write Much.

Think Little.

Author's Ambiguities in Tables of Integrals SUCK!!!

970-290-0550

78. $\int \csc^n(u)\,du = \frac{-1}{n-1}\cot(u)\csc^{n-2}(u) + \frac{n-2}{n-1}\int \csc^{n-2}(u)\,du$

79. $\int \sin(au)\sin(bu)\,du = \frac{\sin((a-b)u)}{2(a-b)} - \frac{\sin((a+b)u)}{2(a+b)} + C$

80. $\int \cos(au)\cos(bu)\,du = \frac{\sin((a-b)u)}{2(a-b)} + \frac{\sin((a+b)u)}{2(a+b)} + C$

81. $\int \sin(au)\cos(bu)\,du = -\frac{\cos((a-b)u)}{2(a-b)} - \frac{\cos((a+b)u)}{2(a+b)} + C$