0/1 points SCalc9 7.6.504.XP. [4597960]

Use the Table of Integrals to evaluate the integral. (Remember to use absolute values where appropriate. Use C for the constant of integration.)

$\int x^6 \operatorname{csch}(x^7 + 3)\, dx = \frac{1}{7} \tan$

$u = x^7 + 3 \Longrightarrow$

$du = 7x^6 dx$

108. $\int \operatorname{csch} u\, du = \ln \left| \tanh \frac{1}{2} u \right| + C$

$= \ln \left| \tanh \left(\frac{1}{2} u\right) \right| + C$

$= \ln \left| \tanh \left(\frac{u}{2}\right) \right| + C$

$\int x^2 \operatorname{sech}(x^3 + 1)\, dx$

$\frac{1}{3} \ln \left| \tanh \left(\frac{u}{2}\right) \right| + C$

$= \frac{1}{3} \ln \left| \tanh \left(\frac{x^3+1}{2}\right) \right| + C$

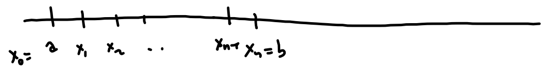
Quinton nailed it.

WebAssign's being an ass.

§7.7 Approximate Integration.

Partition $[a,b]$ into n subintervals of width $\Delta x = \frac{b-a}{n}$

$a = x_0,\ a+\Delta x = x_1,\ a+2\Delta x = x_2,\ \dots,\ a+n\Delta x = x_n$



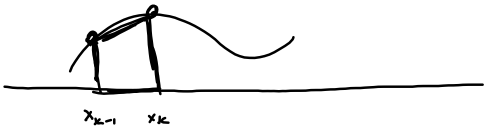
LEFT: $\int_a^b f(x)dx \approx \sum_{k=1}^{n} f(x_{k-1})\Delta x = \sum_{k=1}^{n} f(x_{k-1})\frac{b-a}{n} = \frac{b-a}{n}\sum_{k=1}^{n} f(x_{k-1}) = L_n$

RIGHT $\int_a^b f(x)dx \approx \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} f(x_k)\left(\frac{b-a}{n}\right) = \frac{b-a}{n}\sum_{k=1}^{n} f(x_k) = R_n$

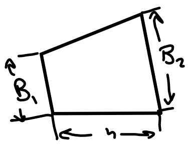
Midpoint: $\bar{x}_1 = \frac{x_0+x_1}{2},\ \bar{x}_2 = \frac{x_1+x_2}{2},\ \dots,\ \bar{x}_n = \frac{x_{n-1}+x_n}{2}$

$\bar{x}_k$ = midpoint of k^{th} subinterval $[x_{k-1}, x_k]$

Trapezoidal: Turns out to be average of L_n & R_n



Area of trapezoid is $\frac{1}{2}(B_1+B_2)h$, where B_1, B_2 are the 2 bases & h is the height



$B_1 = f(x_{k-1})$
$B_2 = f(x_k)$
$h = \Delta x$

$\frac{1}{2}\sum_{k=1}^{n}(B_1+B_2)_k h = \frac{1}{2}\sum\left(f(x_{k-1}) + f(x_k)\right)\Delta x = \frac{1}{2}\left[\sum_{k=1}^{n} f(x_{k-1})\Delta x + \sum_{k=1}^{n} f(x_k)\Delta x\right]$

$= \frac{1}{2}\left[L_n + R_n\right]$! So easy if L_n & R_n already done!

If not, then $T_n = \frac{1}{2}\left[f(x_0) + f(x_1) + f(x_1) + f(x_2) + f(x_2) + f(x_3) + \right.$

$\left. \dots + f(x_{n-2}) + f(x_{n-1}) + f(x_{n-1}) + f(x_n)\right]\Delta x$

$= \frac{1}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right]\Delta x$

$= \frac{\Delta x}{2}\left[f(a) + \sum_{k=1}^{n-1} 2f(x_k) + f(b)\right]$!

FACT: Midpoint Rule is more accurate than L_n, R_n, or T_n

Do L_n, R_n, T_n for $\int_1^7 \sqrt{x^2 - \sin(x)}\,dx$

$n = 5, 10, 50, 100$

$\Delta x = \frac{b-a}{n} = \frac{7-1}{n} = \frac{6}{n}$

$x_k = a + k\Delta x = 1 + k\left(\frac{6}{n}\right)$
$= 1 + \frac{6k}{n} = \frac{6k+n}{n}$

$L_n = \sum_{k=1}^{n} f(x_k)\Delta x = \frac{6}{n}\sum_{k=1}^{n}\sqrt{x_{k-1}^2 - \sin(x_{k-1})}$

$f := x \longrightarrow \sqrt{x^2 - \sin(x)}$

Programming the MidPoint Rule

$$\frac{x_{k-1}+x_k}{2} = \frac{a+(k-1)\left(\frac{b-a}{n}\right) + a + k\left(\frac{b-a}{n}\right)}{2}$$

$$= \frac{2a + k\left(\frac{b-a}{n}\right) - \left(\frac{b-a}{n}\right) + k\left(\frac{b-a}{n}\right)}{2}$$

$$= \frac{2a + 2k\left(\frac{b-a}{n}\right) - \frac{b-a}{n}}{2} = XKBAR$$

$$XKBAR := (k,a,b,n) \longrightarrow$$

$$\frac{1}{2} * \left(2*a + 2*k*\left(\frac{b-a}{n}\right) - \frac{b-a}{n}\right) = k^{th} \text{ input to } f.$$

$$MyMid := \frac{b-a}{n} \cdot \sum_{k=1}^{n} f(XKBAR(k,a,b,n))$$

Error Bounds

E_T = Error from Trapezoidal Rule $\leq \frac{K(b-a)^3}{12n^2}$

E_M = " " Midpoint Rule $\leq \frac{K(b-a)^3}{24n^2}$

where $\boxed{K \geq |f''(x)|}$ $\forall\ x \in [a,b]$

Typically $\exists$ max on $|f''(x)|$ but not necessarily. It depends on continuity of f''.

$f(x) = \sqrt{x}$ @ $x=0$ $\mathcal{D} = [0,\infty)$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\nexists$ @ $x=0$ $\notin$ $f'(x)$ unbounded at the endpoint $x=0$

Improper Integrals.

$\int_0^1 \frac{1}{\sqrt{x}}\,dx$ FTC II requires f cnts on $[0,1]$ for $\int_0^1 f(x)dx = F(1) - F(0)$ for F an antiderivative of f.

$\frac{1}{\sqrt{x}} \nexists$ @ $x=0$

$\Rightarrow f(x) = \frac{1}{\sqrt{x}}$ is not cnts on $[0,1] \Rightarrow$

CAN'T USE FTC II!

However $\int_0^1 \frac{dx}{\sqrt{x}} \exists$. (Book says "converges")

$$\int_0^1 \frac{dx}{\sqrt{x}} = \lim_{a\to 0} \int_a^1 \frac{dx}{x^{1/2}} = \lim_{a\to 0} \int_a^1 x^{-\frac{1}{2}}dx = \lim_{a\to 0} \left[2x^{\frac{1}{2}}\right]_a^1$$

$$= \lim_{a\to 0} \left[2(1)^{\frac{1}{2}} - 2(a)^{\frac{1}{2}}\right] = 2 - 2(0)^{\frac{1}{2}} = \boxed{2}$$

$\int_0^M \frac{dx}{x^n}$ works for $0 < n < 1$

$\int_0^M \frac{dx}{x}$ Diverges! (ie. $\nexists$)

$$\int_1^\infty \frac{dx}{x^n} = \lim_{a\to\infty} \int_1^a x^{-n}dx = \lim_{a\to\infty} \left[\frac{x^{-n+1}}{-n+1}\right]_1^a$$

$$= \lim_{a\to\infty} \left[\frac{a^{-n+1}}{-n+1} - \frac{1^{-n+1}}{-n+1}\right]$$

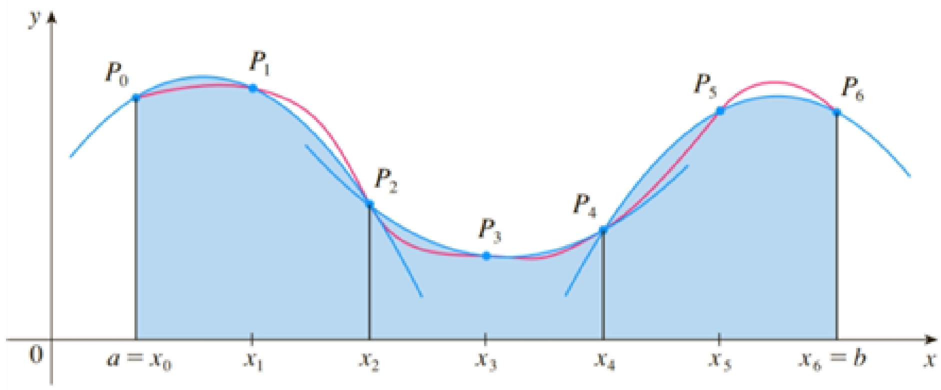
Need $-n+1$ to be negative, so ∞^{-n+1} vanishes

Need $n > 1$ for $\int_1^\infty \frac{dx}{x^n}$ to converge

Open-Ended Writing Project
"programming" Numerical Integration with Maple.

Simpson's Rule

Use parabolas to approximate $f(x)$ between 3 points



Derivation:

Assume middle point is $(0, f(0))$