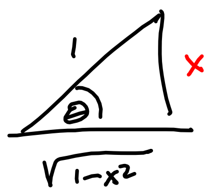
575 #5

7.6 ~~7.5~~ - Use the dad-gum tables in the back of the book!

$$\int_0^{\sqrt{2}/2} \frac{13x^2\,dx}{\sqrt{1-x^2}}$$




$\frac{x}{1} = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$x = \sin\theta$

$\sqrt{1-x^2}$

$= \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta}$

$= |\cos\theta|$

$$= \int_{x=0}^{x=\frac{\sqrt{2}}{2}} \frac{13\sin^2\theta \cdot \cos\theta\, d\theta}{|\cos\theta|}$$

$x = \frac{\sqrt{2}}{2} = \sin\theta \Rightarrow \theta = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

we're keeping things in QI

$x = 0 \Rightarrow \theta = 0$

$$= \int_0^{\frac{\pi}{4}} \frac{13\sin^2\theta\, \cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}} = 13\int_0^{\frac{\pi}{4}} \frac{1-\cos(2\theta)}{2}\, d\theta =$$

$$= \frac{13}{2}\int_0^{\frac{\pi}{4}} (1 - \cos(2\theta))\, d\theta = \frac{13}{2}\int_0^{\frac{\pi}{4}} d\theta - \frac{13}{2}\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}} \cos(2\theta)\cdot 2\, d\theta$$

$u = 2\theta$

$du = 2\, d\theta$

$$= \frac{13}{2}\theta\Big]_0^{\frac{\pi}{4}} - \frac{13}{4}\Big[\sin(2\theta)\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{13}{2}\left[\frac{\pi}{4} - 0\right] - \frac{13}{4}\left[\sin\left(\frac{\pi}{2}\right) - \sin(0)\right]$$

$$= \frac{13\pi}{8} - \frac{13\pi}{4}[1 - 0] = -\frac{13\pi}{8}$$

Negative?! UNPOSSIBLE!

Oh! Idiot! Just $\frac{13}{4}$, so $\boxed{I = \frac{13\pi}{8} - \frac{13}{4}}$

$$\int_0^{\sqrt{2}/2} \frac{13x^2\,dx}{\sqrt{1-x^2}} = \int_{x=0}^{x=\frac{\sqrt{2}}{2}} \frac{13\sin^2\theta\cdot\cos\theta\,d\theta}{|\cos\theta|} = \int_{x=0}^{x=\frac{\sqrt{2}}{2}} 13\sin^2\theta\,d\theta$$

$$\frac{13}{2}\theta\Big]_{x=0}^{x=\frac{\sqrt{2}}{2}} - \frac{13}{4}\Big[\sin(2\theta)\Big]_{x=0}^{x=\frac{\sqrt{2}}{2}} = \frac{13}{2}\arcsin(x)\Big]_0^{\frac{\sqrt{2}}{2}} - \frac{13}{4}\Big[2x\sqrt{1-x^2}\Big]_0^{\frac{\sqrt{2}}{2}}$$

$x = \sin\theta$

$\Rightarrow \theta = \arcsin(x)$ in QI

$\sin(2\theta) =$
$2\sin\theta\cos\theta$
$= 2x\sqrt{1-x^2}$

$$= \frac{13}{2}\left[\arcsin\left(\frac{\sqrt{2}}{2}\right) - \arcsin(0)\right] - \frac{13}{4}\left[2\left(\frac{\sqrt{2}}{2}\right)\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2} - 2(0)\sqrt{1-0^2}\right]$$

$$= \frac{13}{2}\cdot\frac{\pi}{4} - \frac{13}{4}\left[\sqrt{2}\sqrt{1-\frac{1}{2}} - 0\right] = \boxed{\frac{13\pi}{8} - \frac{13}{4}}$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$$

$$\int 3e^{4x+e^{4x}}\,dx$$

$$= 3\int e^{4x}e^{e^{4x}}\,dx =$$

Mine $\frac{3}{4}\int e^{e^{4x}}\cdot 4e^{4x}dx = \frac{3}{4}\int e^{u}du$

Student's $3\int e^{4x}e^{u}\left(\frac{du}{4e^{4x}}\right)$

$= \frac{3}{4}\int e^{u}du$, etc.

$u = e^{4x}$

$\Rightarrow du = 4e^{4x}dx$

$du = 4e^{4x}dx = du$

$dx = \frac{du}{4e^{4x}}$

---

Virtually every error in lecture is from rushing & not writing as thoroughly as I might/should.

99% of 7.5 is pattern recognition and so-called "mathematical intuition."

I'll leave you gentlebeings to it and wait to be asked something until 4:15.

Nothing earth-shattering, here. Just examples in the next lecture, as well, in 7.6.

I want to be here to help, but I don't want to be in your way.

I'll hit "record" when you ask me something, if you ask me something and I remember to hit "record."

Now where did I put my coffee?