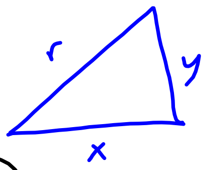
23. $\displaystyle\int \frac{dx}{\sqrt{x^2+2x+5}}$

25. $\displaystyle\int x^2\sqrt{3+2x-x^2}\,dx$

§7.3

Pythagorean-looking stuff

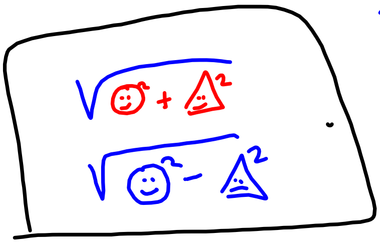


$x^2+y^2=r^2$

$r=\pm\sqrt{x^2+y^2}$

$y^2=r^2-x^2$

$y=\pm\sqrt{r^2-x^2}$



(23) $\displaystyle\int \frac{dx}{\sqrt{x^2+2x+5}}$

$a=1, b=2, c=5$

$b^2-4ac = 2^2-4(1)(5)$

$=4-20=-16$

So, $x^2+2x+5 \neq 0$ $\forall x \in \mathbb{R}$

x^2+2x+5

$= x^2+2x+1^2-1^2+5$

(scratch $\frac{2}{2}=1 \rightsquigarrow 1^2=1$)

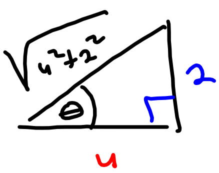
$= (x+1)^2+4$

So #23 is $\displaystyle\int\frac{dx}{\sqrt{(x+1)^2+4}}$

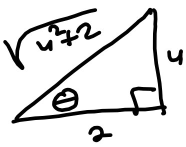
Let $u = x+1$

$du = dx$

$\#23 = \displaystyle\int \frac{du}{\sqrt{u^2+2^2}}$



OR



$\frac{u}{2}=\cot\theta$

$u=2\cot\theta$

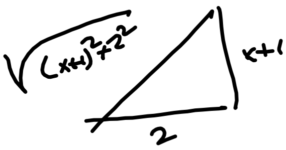
is totally legit.

$\frac{u}{2}=\tan\theta$

$u=2\tan\theta$

is nicer, though. Use this.

Skip the $u=x+1$



$\frac{x+1}{2}=\tan\theta$

$x+1=2\tan\theta$

$x=2\tan\theta-1 \longrightarrow dx=2\sec^2\theta\,d\theta$

So #23 looks like

$\displaystyle\int\frac{2\sec^2\theta\,d\theta}{\sqrt{2^2\tan^2\theta+2^2}}$

$=\displaystyle\int\frac{2\sec^2\theta\,d\theta}{2|\sec\theta|}$

etc.

$$\int \frac{du}{\sqrt{u^2+2^2}} = \int \frac{du}{\sqrt{(2\tan\theta)^2+2^2}}$$?

$$= \int \frac{du}{\sqrt{2^2\tan^2\theta+2^2}}$$ convert!

$$= \int \frac{du}{\sqrt{2^2(\tan^2\theta+1)}}$$

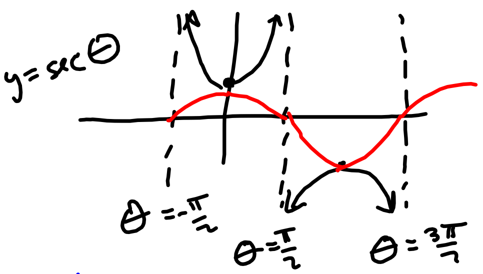
$u = 2\tan\theta \Rightarrow$

$du = 2\sec^2\theta \, d\theta$

$$= \int \frac{du}{2\sqrt{\tan^2\theta+1}}$$

$$= \int \frac{du}{2\sqrt{\sec^2\theta}}$$

$$= \int \frac{du}{2|\sec\theta|} = \int \frac{2\sec^2\theta \, d\theta}{2|\sec\theta|}$$



I'm not seeing how to get rid of the absolute value $|\sec\theta|$

x^2+2x+5 w/o manipulating?

?

For Now, Assume $\sec\theta > 0$

Then #23 is

$$\int \frac{\sec^2\theta\, d\theta}{\sec\theta} = \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta| + C$$

$$= \ln\left|\frac{2}{\sqrt{u^2+4}}\right| + C$$

$$= \ln\left|\frac{2}{\sqrt{(x+1)^2+4}}\right| + C$$

$$= \ln|2| - \ln\left|\sqrt{(x+1)^2+4}\right| + C$$

$$= -\ln\sqrt{(x+1)^2+4} + \hat{C}$$

$\sqrt{u^2+2^2}$, u, 2, θ

what if $|\sec\theta| = -\sec\theta$

Then

$$-\int \sec^2\theta\, d\theta$$

$$= -\ln\left|\frac{2}{\sqrt{(x+1)^2+4}}\right| + C$$

$$= \dots = \ln\sqrt{(x+1)^2+4} + \hat{C}$$

Book says

$$\ln\left|\sqrt{x^2+2x+5} + x + 1\right| + C$$

$$3/4x \quad \frac{3}{4}x = \frac{3x}{4} \quad \frac{3}{4x} \quad ?$$

From Jocelyn Sailas to Me (Direct Message) 03:37 PM

I did u=(x+1) and du=1 then later after you get to the integral of sec(v)dv= ln abs value (tan(v)+sec(v)) and I have v=arctan(1/2u)

dx

so then it was ln abs value(tan(arctan(1/2(x+1))) + sec(arctan(1/2(x+1))))

ambiguous

does that help??

also I have to go. see you on friday!!

23. $\int \frac{dx}{\sqrt{x^2+2x+5}}$

25. $\int x^2\sqrt{3+2x-x^2}\,dx$

25

$-x^2+2x+3$

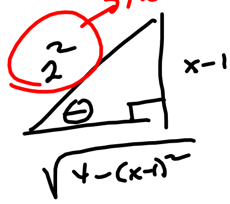
$= -(x^2-2x+1^2)+3+1$

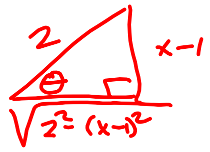
$= -(x-1)^2+4 = 4-(x-1)^2$

No. Just $r=2$

$\Rightarrow$ #25 $\int x^2\sqrt{4-(x-1)^2}\,dx$

says this is r^2





So $\frac{x-1}{2}=\sin\theta$

So $x-1=2\sin\theta$

$x=2\sin\theta+1$

$dx=2\cos\theta\,d\theta$

#25 $\int(2\sin\theta+1)^2\sqrt{2^2-2^2\sin^2\theta}\cdot 2\cos\theta\,d\theta$

$=\int(4\sin^2\theta+4\sin\theta+1)\,2\sqrt{1-\sin^2\theta}\cdot 2\cos\theta\,d\theta$

$=\int(4\sin^2\theta+4\sin\theta+1)(2)(2)|\cos\theta|\cos\theta\,d\theta$

Make the case that $0\le\theta\le\frac{\pi}{2}$

$=4\int(4\sin^2\theta\cos^2\theta+4\sin\theta\cos^2\theta+\cos^2\theta)\,d\theta$

$=4\int 4\left(\frac{1-\cos(2\theta)}{2}\right)\left(\frac{1+\cos(2\theta)}{2}\right)d\theta$

$+\ -\ 4\int 4\cos^2\theta(-\sin\theta)\,d\theta$

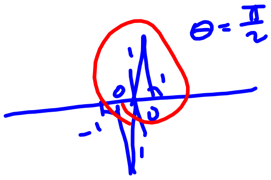
$+\ 4\int\frac{1+\cos(2\theta)}{2}\,d\theta$

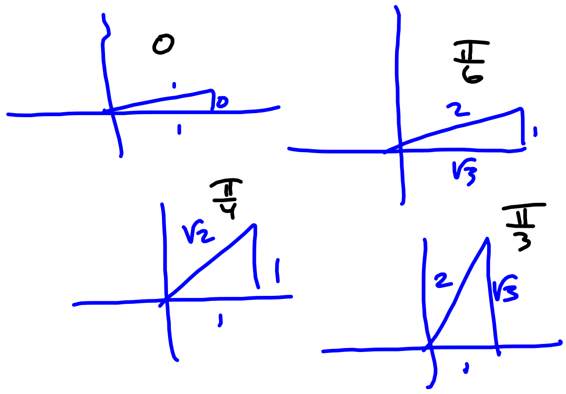
$$\int_0^{\frac{5\pi}{4}} |\cos\theta|\, d\theta =$$

Need $\cos\theta \geq 0$

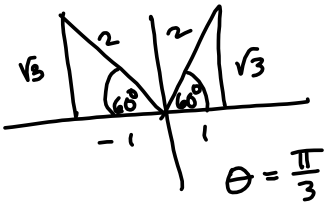
Find $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$



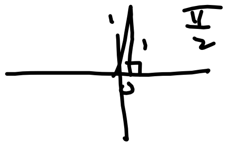


$\sin\theta = \frac{\sqrt{3}}{2}$

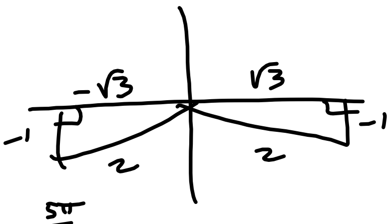


$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$= 60°, 120°$



$\sin\theta = -\frac{1}{2}$



$$\int_0^{\frac{5\pi}{4}} |\cos\theta|\, d\theta = \int_0^{\frac{\pi}{2}} \cos\theta\, d\theta + \int_{\frac{\pi}{2}}^{\frac{5\pi}{4}} -\cos\theta\, d\theta$$