Section 7.1 Integration by Parts

Recall $(fg)' = f'g + fg'$ sledgehammer $\int xe^x dx$

$\implies \int (fg)' = \int (f'g + fg')$

$fg = \int f'g + \int fg'$

$fg - \int f'g = \int fg'$

$\int fg' = fg - \int f'g$

$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

$\int u\,dv = uv - \int v\,du$

$u = f(x)$, $dv = g'(x)dx$

$\int xe^x dx$

∃ formula for $\int x^n e^x dx$

$\left(\text{Let} \quad \begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array} \right)$

$= uv - \int v\,du = xe^x - \int e^x dx$

$= xe^x - e^x + C$, until all the x's go away.

Try one:

$\int \ln(x)\,dx = ?$
Meh.

$$I = \int x \ln(x)\,dx$$

$$\left(\begin{array}{ll} u = \ln(x) & dv = x\,dx \\ du = \frac{1}{x}\,dx & v = \frac{1}{2}x^2 \end{array} \right)$$

$$\Rightarrow I = uv - \int v\,du = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{2}x^2 \ln(x) - \frac{1}{2}\int x\,dx$$

$$= \frac{1}{2}x^2 \ln(x) - \frac{1}{2}\cdot\frac{1}{2}x^2 + C$$

$$= \boxed{\frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + C}$$

$\int \ln(x)\,dx$ You can integrate $\ln(x)$ with Integration by Parts.

$u = \ln(x)$ $\quad dv = dx$!

$I = \int e^x \sin(x)\,dx$

$u = e^x \qquad dv = \sin(x)\,dx$

$du = e^x\,dx \qquad v = -\cos(x).$

$\Rightarrow uv - \int v\,du = -e^x\cos(x) + \int \cos(x) e^x\,dx = -e^x \cos(x) + I_2$

$u = e^x \qquad dv = \cos(x)\,dx$

$du = e^x\,dx \qquad v = \sin(x)$

Back-filling missing "C"

$\Rightarrow I_2 = uv - \int v\,du = e^x \sin(x) - \int e^x \sin(x)\,dx \; \hat{+C} = e^x \sin(x) - I + \hat{C}$

$\Rightarrow I = -e^x \cos(x) + e^x \sin(x) - I + \hat{C}$

$+I \qquad = \qquad +I$

$2I = e^x \sin(x) - e^x \cos(x) + \hat{C}$

$$\boxed{I = \frac{e^x \sin(x) - e^x \cos(x)}{2} + C, \text{ where } C = \frac{\hat{C}}{2}}$$

Oops! Missing my "C"!

$\sin(3\theta) \neq \sin(3)\theta = (\sin(3))(\theta)$

So, $\sin 3\theta$ is ambiguous!