All systems Go!

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\cosh x) = \sinh x \qquad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$$

One like #5c on WP#1

$$f(x) = \log_7(\sin(x)) = \frac{1}{\ln(11)} \ln(\sin(x)) \Rightarrow f'(x) = \frac{1}{\ln(11)}\left(\frac{\cos(x)}{\sin(x)}\right)$$

$$= \boxed{\frac{1}{\ln(11)} \cot(x)}$$

Change of base

$$\log_{11}(x) = \frac{\ln(x)}{\ln(11)}$$

Most calculators don't have hypebolic trig functions, but you can build them out of other functions, at least on the homework.

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

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 Plot1 Plot2 Plot3
\Y1=(e^(X)-e^(-X
))/2
\Y2=(e^(X)+e^(-X
))/2
\Y3=Y1/Y2
\Y4=
\Y5=
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$$\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right) \qquad x \in \mathbb{R}$$

Let's Prove This!

$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\right) \qquad x \geqslant 1$$

$$\tanh^{-1}x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -1 < x < 1$$

$f(x) = y = \sinh(x)$. we find $y^{-1} = f^{-1}(x)$

How do you find an inverse:

Swap x & y. Solve for y.

$$x = \sinh(y) = \frac{e^y - e^{-y}}{2} = x$$

$$\Rightarrow e^y - e^{-y} = 2x \Rightarrow$$

$$e^y - e^{-y} - 2x = 0$$

$$e^{-y}\left[e^y e^y - e^{-y}e^y - 2xe^y\right] = 0, \text{ b/c } \frac{m}{e^{-y}} = e^y m$$

$$\Rightarrow e^{-y} = 0 \text{ OR } e^{2y} - 2xe^y - 1 = 0$$

$$(e^y)^2 - (2x)e^y - 1 = 0$$

$$u^2 - (2x)u - 1 = 0$$

$a = 1, b = -2x, c = -1$

$$\Rightarrow b^2 - 4ac = (-2x)^2 - 4(1)(-1) = 4x^2 + 4 = 4(x^2+1)$$

$$\Rightarrow u = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2x \pm \sqrt{4(x^2+1)}}{2} = \frac{2x \pm 2\sqrt{x^2+1}}{2}$$

$$= x \pm \sqrt{x^2+1} = u = e^y \Rightarrow$$

$$0 < e^y = x \pm \sqrt{x^2+1} \Rightarrow$$

$$y = \ln\left(x \pm \sqrt{x^2+1}\right)$$

~~$\Rightarrow$ Need $2x \pm \sqrt{x^2+1} > 0$~~

~~$0 < 2x + \sqrt{x^2+1}$ will be fine~~

~~But $2x - \sqrt{x^2+1} = 4 - \sqrt{5}$ is OK~~

~~$x = 2$: $4 - \sqrt{4+1}$~~

~~$x = 4$: $8 - \sqrt{16+1} = 8 - \sqrt{17}$~~

~~$x = 100$: $200 - \sqrt{100^2+1}$~~

Victoria

You messed-up your ALGEBRA.

Can we make an argument for the choice of "+" over "−"?

No, b/c your algebra sucks.

$$e^y = x \pm \sqrt{x^2+1}$$

$e^y > 0$ & $x - \sqrt{x^2+1} < 0 \ \forall x > 0$.

So $e^y = x + \sqrt{x^2+1}$ is the ONLY Sol'n

& it works $\forall x \in \mathbb{R}$.

$x = -500$

$$x + \sqrt{x^2+1} = -500 + \sqrt{500^2+1} > -500 + \sqrt{500^2}$$

$$= -500 + 500 = 0$$