14. 0/1 points

Evaluate the integral. (Use C for the constant of integration.)

$$\int \frac{dx}{\sqrt{3x}\,(1+3x)}$$

$C + \frac{2}{3}\tan^{-1}\left(\sqrt{3}\sqrt{x}\right)$

Rebooted after class and everything works perfectly.

2 methods:

(1) Mills's bludgeon

(2) Chanthalangsy's dagger.

(1) Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}\,dx \Rightarrow dx = 2x^{\frac{1}{2}}du = 2\sqrt{x}\,du$

$$\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{x}(1+3x)} = \frac{1}{\sqrt{3}}\int \frac{2\sqrt{x}\,du}{\sqrt{x}(1+3u^2)} = \frac{2}{\sqrt{3}}\int \frac{du}{1+3u^2}$$

$$= \frac{2}{\sqrt{3}}\int \frac{du}{1+(\sqrt{3}u)^2} = \frac{2}{\sqrt{3}}\int \frac{\frac{1}{\sqrt{3}}dv}{1+v^2} = \frac{2}{3}\int \frac{dv}{1+v^2} = \frac{2}{3}\tanh^{-1}(v) + C$$

Let $v = \sqrt{3}u \Rightarrow$
$dv = \sqrt{3}\,du \Rightarrow$
$du = \frac{1}{\sqrt{3}}dv$

$$= \frac{2}{3}\tanh^{-1}(\sqrt{3}u) + C$$
$$= \frac{2}{3}\tanh^{-1}(\sqrt{3}\sqrt{x}) + C$$

(2) Let $u = \sqrt{3x} = \sqrt{3}\sqrt{x}$

$\Rightarrow du = \sqrt{3}\cdot\frac{1}{2\sqrt{x}}\,dx \Rightarrow dx = \frac{2\sqrt{x}}{\sqrt{3}}du$ & $3x = (\sqrt{3x})^2 = u^2$

$$= \int \frac{\frac{2\sqrt{x}}{\sqrt{3}}du}{\sqrt{3}\sqrt{x}(1+u^2)} = \int \frac{2\sqrt{x}\,du}{\sqrt{3}\sqrt{3}\sqrt{x}(1+u^2)} = \frac{2}{3}\int \frac{du}{1+u^2}$$

$$= \frac{2}{3}\tanh^{-1}(u) + C$$

$= \frac{2}{3}\tanh^{-1}(\sqrt{3}\sqrt{x}) + C$. Much slicker.

I didn't see it, at first, but I did have a hammer & helpers.

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

12. 0/1 points SCalc9 6.8.0

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \to 0} \sin(7x)\csc(3x)$$

✗ $\frac{7}{3}$

$$\frac{\sin(7x)}{\sin(3x)} \xrightarrow[L'H]{x\to 0} \frac{7\cos(7x)}{3\cos(3x)} \xrightarrow{x\to 0} \frac{7\cdot 1}{3\cdot 1} = \frac{7}{3}$$

$$\frac{7x^5 + lower - \deg ree\, terms}{11x^5 + lower - \deg ree\, terms} \xrightarrow{x\to\infty} \frac{7x^5}{11x^5}$$

Hat tip to Quinton for his work on 6.6!!!

Section 6.7 Stufrf

11. 0/1 points SCalc9 6.7.052.

Find the derivative.

$$y(x) = x\tanh^{-1}(x) + \ln\left(\sqrt{1-x^2}\right)$$

6 Derivatives of Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$$

$$y' = \tanh^{-1} + x\cdot\frac{1}{1-x^2} + \frac{1}{2}\cdot\frac{-2x}{1-x^2}, \text{ using}$$

$$\ln\left(\sqrt{1-x^2}\right) = \ln\left(\left(1-x^2\right)^{\frac{1}{2}}\right) = \frac{1}{2}\ln\left(1-x^2\right)$$