Today, 6.6 - Inverse Trig Functions, maybe a little 6.7.

Questions from any topic...

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

Here's a pretty cool, pretty unexpected identity!

$f(x) = $

EXAMPLE 6 Prove the identity $\tan^{-1}x + \cot^{-1}x = \pi/2$.

Note $\frac{d}{dx}[\arctan(x) + \operatorname{arccot}(x)]$

$= \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0 = \frac{d}{dx}[f(x)]$

$\Rightarrow f(x) = \text{constant}$

Note $\arctan(1) + \operatorname{arccot}(1) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ ■

40. Find an equation of the tangent line to the curve $y = 3\arccos(x/2)$ at the point $(1, \pi)$.

$\left(\frac{d}{dx}[\arccos(x)] = -\frac{1}{\sqrt{1-x^2}} \right.$

$\left. \frac{d}{dx}[\arccos(u)] = -\frac{1}{1-u^2} \cdot \frac{du}{dx} \right)$

$\Rightarrow \frac{d}{dx}\left[3\arccos\left(\frac{x}{2}\right)\right] = -\frac{3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}$

$\left(u = \frac{x}{2} \Rightarrow u' = \frac{1}{2}\right)$

$y - y_0 = m(x - x_0)$

$y = m(x - x_0) + y_0$ $\quad y = m(x - x_1) + y_1$

$\Rightarrow y = f'(x_0)(x - x_0) + f(x_0)$

Scratch:

$f'(1) = \frac{-3}{2\sqrt{1-\left(\frac{1}{2}\right)^2}}$

$= \frac{-3}{2\sqrt{\frac{3}{4}}} = \frac{-3}{2 \cdot \frac{\sqrt{3}}{2}} = \frac{-3}{\sqrt{3}}$

$= \frac{-3\sqrt{3}}{3} = \boxed{-\sqrt{3} = f'(1)}$

written homeworks:

(40) we find an eq'n of the tangent line to $f(x) = 3\arccos\left(\frac{x}{2}\right)$ @ $(x_0, y_0) = (1, \pi)$:

$f'(x) = \frac{-3}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$ $\Rightarrow$

tangent line is

$y = m(x - x_0) + f(x_0)$

$= f'(x_0)(x - x_0) + f(x_0)$

$= f'(1)(x-1) + f(1)$

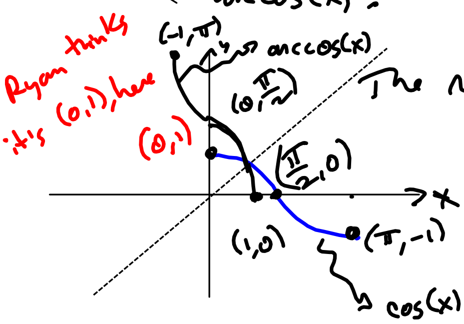
$= \boxed{-\sqrt{3}(x-1) + \pi} = y$

fine for me!

WebAssign probably wants it simplified to $y = mx + b$

$\boxed{y = -\sqrt{3}\,x + \sqrt{3} + \pi}$

arccos(x) :



The restricted cosine func.

The graph of arccos(x) is ——

″ ″ ″ cos(x) is ——

$\mathcal{D}(\cos(x))^* = [0,\pi] = \mathcal{R}(\arccos(x))$

$\mathcal{R}(\cos(x))^* = [-1,1] = \mathcal{D}(\arccos(x))$

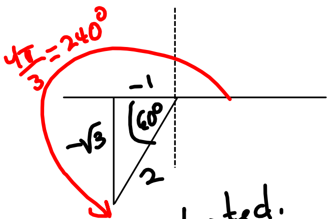
*Restricted to be 1-to-1

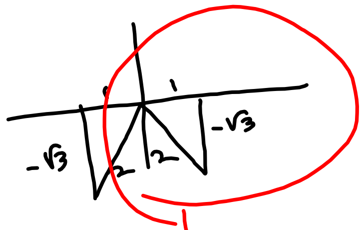
Be careful!

$f^{-1}(f(x)) = x$

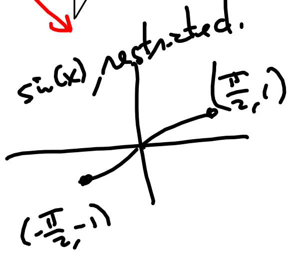
$f(f^{-1}(x)) = x$

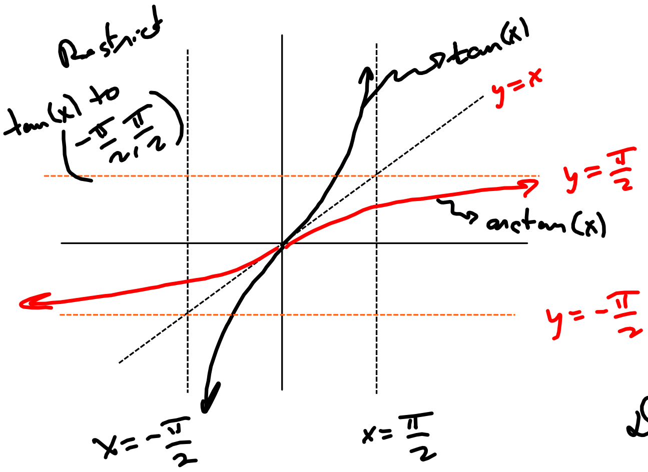
But $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$





This is the one $\sin^{-1}(x)$ $= \arcsin(x)$ sees & it thinks it's $x = -\frac{\pi}{3} = -60°$





$$\frac{d}{dx}\left[\tan(x)\right]\Big|_{x=0}$$

$$= \sec^2(x)\Big|_{x=0} = 1$$

¢ concave up for $x \in (0, \frac{\pi}{2})$

¢ " down " " " $(-\frac{\pi}{2}, 0)$

$\mathcal{D}(\tan(x))^* = (-\frac{\pi}{2}, \frac{\pi}{2}) = \mathcal{R}(\arctan(x))$

$\mathcal{R}(\tan(x))^* = (-\infty, \infty) = \mathcal{D}(\arctan(x))$

Simple Interest

$A = P + I = P + Prt = P(1+rt)$

Compound Interest

$m = \#$ of periods per year

r = annual rate of interest.

$i = \frac{r}{m}$

t = time

$n = \#$ of periods $= mt$

P = Principle

A = Future Value

Period

0 $\quad P$

1 $\quad P + Pi = P(1+i) =$ New "P"

2 $\quad P + Pi + (P+Pi)i = P(1+i) + P(1+i)i = P(1+i)(1+i) = P(1+i)^2$
$= P(1+i)^2$

$\vdots$

n $\quad P(1+i)^n = P\left(1+\frac{r}{m}\right)^{mt} = P\left(1+\frac{r}{m}\right)^{\frac{m}{r}(rt)}$

$$= P\left(1 + \frac{1}{\frac{m}{r}}\right)^{\left(\frac{m}{r}\right)(rt)}$$

$$= P\left(\left(1 + \frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{rt}$$

$m \to \infty \longrightarrow$ Continuous Compounding

$$A = P\left((1+i)^{\frac{1}{i}}\right)^{rt} = P\left(\left(1+\frac{1}{u}\right)^{u}\right)^{rt}$$

$\frac{r}{m} = i \quad i \to 0 \qquad m \to \infty$

Now, $m \to \infty$ means $\frac{r}{m} = i \longrightarrow 0$

$$\lim_{i \to 0} (1+i)^{\frac{1}{i}} = \lim_{i \to 0} B \text{, when } B = (1+i)^{\frac{1}{i}}$$

$$B = (1+i)^{\frac{1}{i}}$$

$$\ln(B) = \ln\left((1+i)^{\frac{1}{i}}\right) = \frac{1}{i}\ln(1+i) \xrightarrow{i \to 0} \frac{1}{0}\ln(1)$$
$= \infty \cdot 0$

Can't do it easily w/o S6.8! $\quad$ L'Hôpitals!

S6.8 If $\frac{f}{g} \xrightarrow{x \to *} \frac{0}{0}$ or $\frac{\infty}{\infty}$ or 1^{∞} or $0 \cdot \infty$,

then L'Hôpital can help.

Take derivative of Top & Bottom

$\frac{\ln(1+i)}{i} \xrightarrow{i \to 0} \frac{0}{0}$ is L'Hôpital territory

$$\frac{\frac{d}{di}[\ln(1+i)]}{\frac{d}{di}[i]} = \frac{\frac{1}{1+i}}{1} = \frac{1}{1+i} \xrightarrow{i \to 0} \frac{1}{1} = 1$$

$\therefore \lim_{i \to 0} \ln(B) = \lim_{i \to 0} \ln\left((1+i)^{\frac{1}{i}}\right) = 1$

$\therefore \lim_{i \to 0} \left(e^{\ln(B)}\right) = e^1$

$\therefore \lim_{i \to 0} B = \lim_{i \to 0} \left((1+i)^{\frac{1}{i}}\right) = e$

$\therefore \lim_{i \to 0} P\left((1+i)^{\frac{1}{i}}\right)^{rt} = Pe^{rt}$

Try it for

$$A = P\left(1+\frac{r}{m}\right)^{mt} = P\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}(rt)}$$

$$P\left(\left(1+\frac{1}{u}\right)^{u}\right)^{rt} \xrightarrow{u \to \infty} ?$$

where $u = \frac{m}{r}$

§6.7 Hyperbolic Trig Functions

$\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\quad$ $\operatorname{csch}(x) = \frac{1}{\sinh(x)}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$ $\quad$ $\operatorname{sech}(x) = \frac{1}{\cosh(x)}$

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ $\quad$ $\coth(x) = \frac{1}{\tanh(x)}$

Zach notices this is also different from circular trig functions

$\frac{d}{dx}(\sinh x) = \cosh x$ $\quad$ $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

$\frac{d}{dx}(\cosh x) = \sinh x$ $\quad$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ Only

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$ $\quad$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$

$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$

?!