6.6 by the end of the day, today...

6.5 and questions to kick things off...

$f(t) = Ce^{kt}$ & k is either positive or negative

$\frac{df}{dt} = kf$ for some Konstant K.

$K > 0$ Growth

$K < 0$ Decay

My way.

The book way:

Ce^{-kt} $\quad \frac{dP}{dt} = -KP$ for decay

Ce^{kt} $\quad \frac{dP}{dt} = KP$ for growth

But K is always positive

S5.5 #8

8. 0/1 points SCalc9 5.5.017.MI. [4934571]

In a certain city the temperature (in °F) t hours after 9 a.m. was modeled by the function

$$T(t) = 40 + 19\sin\left(\frac{\pi t}{12}\right).$$

Find the average temperature T_{ave} (in °F) during the period from 9 a.m. to 9 p.m. (Round your answer to the nearest degree.)

Lexicon for all word problems:

Let t = time in hours after 9 a.m.

$T = T(t)$ = Temperature as a function of t

Want T_{avg} = average temp between 9am and 9pm

$\Rightarrow t \in [0, 12]$

time = 9a $\Rightarrow t = 9-9 = 0$

time = 9p $\Rightarrow t = 21 - 9 = 12$

9pm = 21st hour.

$$T_{AVG} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{12-0}\int_0^{12}\left(19\sin\left(\frac{\pi t}{12}\right) + 40\right)dt$$

etc.

§6.5 Exponential Growth & Decay

Basic Idea:

Growth rate is proportional to size

P = Population*, Learning,

* Like bacteria

t = time (seconds, minutes, hours, days, years, ...)

$\propto$

$\frac{dP}{dt} \propto P$ means proportional.

$\frac{dP}{dt} = kP$ for some fixed k.

Relative Growth Rate is Constant

$$\frac{1}{P}\frac{dP}{dt} = \frac{\frac{dP}{dt}}{P} = k$$

Recall error estimates?

Error = P is $\pm .1$ cm, $P = 30$

Then relative error is $\frac{\pm .1}{30} = \frac{\text{Error}}{\text{How Big it is.}}$

percentage error:

$$\left(\frac{\pm .1}{30}\right)(100\%)$$

All such equations for P give rise to the exponential model

$P = P(t) = Ce^{kt}$ (k = relative growth rate)

$$\frac{dP}{dt} = kP \Longrightarrow$$

$$\frac{dP}{P} = k\,dt$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{dP}{P} = \int k\,dt = k\int dt$$

$\ln|P| + \hat{C} = kt + \hat{\hat{C}}$ Let $\tilde{C} = \hat{\hat{C}} - \hat{C}$

$$\Longrightarrow \ln|P| = kt + \tilde{C}.$$

Assume Pop > 0

Then $\ln P = kt + \tilde{C} \Longrightarrow$

$$e^{\ln P} = e^{kt+\tilde{C}}$$

$P = e^{kt+\tilde{C}} = e^{kt}e^{\tilde{C}}$ Let $e^{\tilde{C}} = C$

$$\Longrightarrow \boxed{P = Ce^{kt}}$$

$\Longrightarrow P = Ce^{kt} \Longrightarrow C = P(0)$

since $P(0) = ce^{k\cdot 0} = ce^0 = C$

Initial Value is important!

Note: $(a^b)^c = a^{bc}$

$$3^t = \left(e^{\ln(3)}\right)^t = e^{(\ln(3))t}$$

THE $\frac{1}{2}$-Life Equation

"The half-life of C-14 is approx. 5700 yrs."

Write the exponential decay model for C-14

$$\boxed{P(t) = P_0 e^{kt}}$$

$P(5700) = \frac{1}{2}P(0)$

$P_0 e^{5700k} = \frac{1}{2}P_0$

$e^{5700k} = \frac{1}{2}$ solve for k

$$\ln\left(e^{5700k}\right) = \ln\left(\tfrac{1}{2}\right)$$

$5700k = \ln\left(\frac{1}{2}\right)$

$k = \dfrac{\ln(1/2)}{5700} = -\dfrac{\ln(2)}{5700}$

So $P(t) = P_0 e^{-\frac{\ln(2)}{5700}t}$

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14*3600/3/5280
      3.181818182
Ans►Frac
            35/11
ln(2)/5700
   1.21604769E-4
```

$-\dfrac{\ln(2)}{5700} \approx -1.21604769 \times 10^{-4}$

OR $\boxed{-.000121604769 \approx k}$

& use "k" until you need the actual #.

Newton's Law of Cooling

T_0 = ambient temperature (The room!)

T = Temperature as a function of

t = time, in (say) hours.

Newton says: $T'(t) = k(T - T_0)$ $T(t)$

Make the substitution $P = T - T_0$ $(P' = T')$

Then $P'(t) = kP.$ $P(t)$

§6.6 Inverse Trig Functions

$y = \sin^{-1}(x) = \arcsin(x)$

Arithmetic Powers

$$\sin^2(x) = (\sin(x))(\sin(x))$$

$$\sin^{-3}(x) = \frac{1}{\sin^3(x)}$$

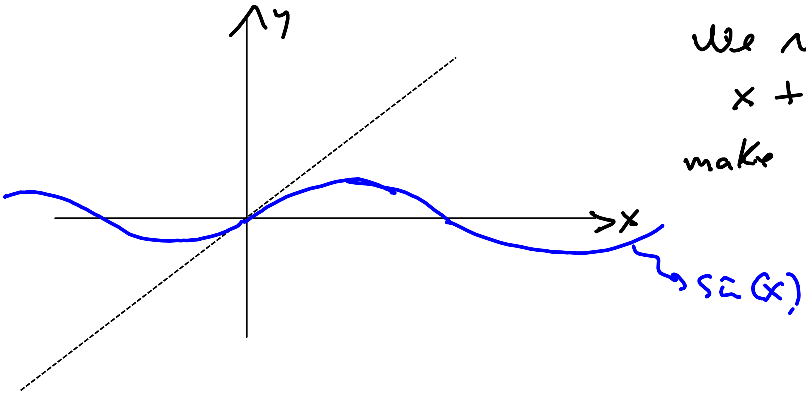
$\sin^{-1}$ is inverse with respect to function composition, not arithmetic

$\sin^{-1}(x)$ is the <u>function</u> inverse of sin. Sucks.

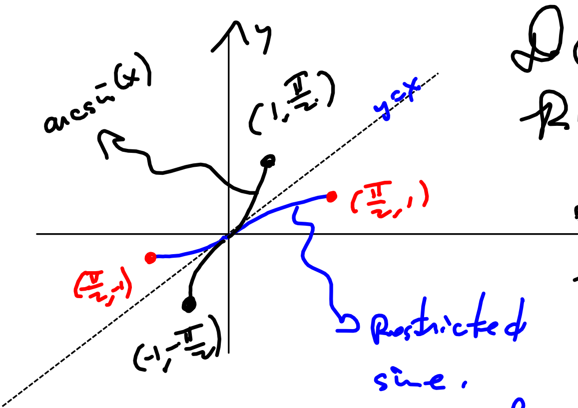
$\sin^{-1}(x)$ is NOT $\frac{1}{\sin(x)}$

$$\sin(\sin^{-1}(x)) = x$$

$$\sin^{-1}(\sin(x)) = x$$



we restrict x to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to make $\sin(x)$ 1-to-1.



$\mathcal{D}(\text{restricted sine}) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\mathcal{R}(\quad " \qquad " \quad) = [-1, 1]$

$\mathcal{D}(\arcsin(x)) = [-1, 1]$

$\mathcal{R}(\arcsin(x)) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

sine is cnt^s & difb^l $\longrightarrow$

arcsine " " " "

So let's talk about $\frac{d}{dx}[\arcsin(x)]$

2 methods:

METHOD #1 FORMULA FOR Derivatives of inverse functions

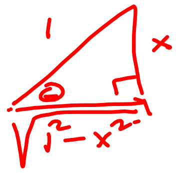
$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}$$

$f^{-1}(x) = \arcsin(x)$

$\Rightarrow f(x) = \sin(x)$

$f'(x) = \cos(x)$

What's $\cos(\arcsin(x)) = \cos(\theta)$



$$\therefore \frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$$

METHOD #2

$y = \arcsin(x) \longrightarrow$

$\sin(y) = \sin(\arcsin(x))$ (for x in the (a) right domain)

$\frac{d}{dx}\left[\ \sin(y) = x\ \right]$ Differentiate Implicitly.

$\cos(y)\,\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos(y)} = \sec(y) = \sec(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$