Things I'm building this weekend:

Writing Projects. 4 of them. They're like take-home tests.

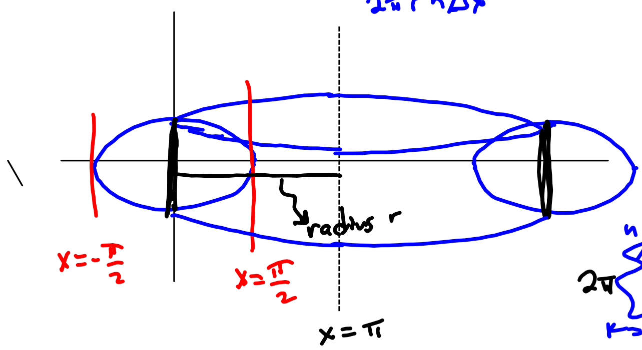
E-Mail Settings Assignment.

All that stuff will live on D2L Assignments link.

§5.3 : Volume of the solid obtained by revolving

$y = \cos^4(x)$ & $y = -\cos^4(x)$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, about $x = \pi$

$2\pi r h \Delta x$



height $= \cos^4(x) - (-\cos^4(x))$

$= 2\cos^4(x)$

$r = \pi - x$

Thickness $= dx = \Delta x$

$$2\pi \sum_{k=1}^{n} (\pi - x) 2\cos^4(x) \Delta x$$

$$\xrightarrow{n \to \infty} 4\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\pi - x)\cos^4(x)\, dx$$

Suggest:

$\cos^4(x) =$

$$\left(\cos^2(x)\right)^2 = \left(\frac{1+\cos(2x)}{2}\right)^2 = \frac{1 + 2\cos(2x) + \cos^2(2x)}{4}$$

$$= \frac{1}{4} + \frac{\cos(2x)}{4} + \frac{1}{4}\left(\frac{1+\cos(4x)}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x)$$

$$= \frac{3}{8} + \frac{1}{4}\cos(2x) + \frac{1}{8}\cos(4x)$$

Work = Force times Distance.

In 5.4, we learn how to handle a variable force, using Calculus.

$W = F \cdot D = F(x) D = W$

6. 0/2 points SCalc9 5.4.009. [4934598]

Suppose that 2 J of work is needed to stretch a spring from its natural length of 36 cm to a length of 47 cm.

(a) How much work (in J) is needed to stretch the spring from 41 cm to 45 cm? (Round your answer to two decimal places.)

0.93 J

(b) How far beyond its natural length (in cm) will a force of 25 N keep the spring stretched? (Round your answer one decimal place.)

7.6 cm

Hooke's Law: $F = kx$, for some konstant k.

Natural Length is 36 cm $\longrightarrow x = 0$

work done to stretch spring from 36 cm to 47 cm is 2 J

$\hookrightarrow 47 - 36 = 11$ cm

$1\,J = 1\,N\text{-}m = 1\,\frac{kg\text{-}m}{s^2} \cdot 1\,m = 1\,\frac{kg\text{-}m^2}{s^2}$

So $\int_0^{11} kx\,dx = 2$ $\longrightarrow$ 0.11 cm

$(.11)(.11) = ~~0.0121~~$

$= .0121$

$\int_0^{11} kx\,dx = k\left[\frac{x^2}{2}\right]_0^{11} = \frac{121}{2}k = 2 \Rightarrow$ $.11$ $.0121$

$\boxed{k = \frac{4}{121}}$ ≈ 330.5785124 $.0121$

$\frac{4}{.0121} = \frac{4}{\frac{121}{10000}} = \frac{40000}{121}$

330.5785124

use this!

(b) $F = kx = 25, \Rightarrow$

$x = \frac{25}{k} = \frac{25}{\frac{4}{121}} = 25\left(\frac{121}{4}\right)$ $\frac{40000}{121}$ $\frac{121}{40000}$

Ryan says "Steve, you're mixing MKS & CGS systems!"

(you idiot).

We need to convert cm to m.

So let's do it right: $k = \frac{40000}{121}$

$x = \frac{25}{k} = \frac{25}{\frac{40000}{121}} = \frac{25(121)}{40000}$ Takes us in the wrong direction.

Re-Start

$\int_0^{.11} kx\,dx = k\left[\frac{x^2}{2}\right]_0^{.11} = k\left[\frac{.0121}{2}\right] = \frac{121}{20000}k = 2$

$\Rightarrow k = 2\left(\frac{20000}{121}\right) = \boxed{\frac{40000}{121} = k}$

(b) $F = kx = \frac{40000}{121}(x) = 25$

$x = \frac{25(121)}{40000} \approx .075625\ m = 7.56\ cm$

10. 0/4 points SCalc9 5.4.019 [4934615]

A leaky 10-kg bucket is lifted from the ground to a height of 16 m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 48 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 16-m level. How much work is done? (Use 9.8 m/s^2 for g.)

Show how to approximate the required work (in J) by a Riemann sum. (Let x be the height in meters above the ground. Enter x_i^* as x_i.)

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\square \; \boxed{9.8(70.8 - 3.8x_i)} \right) \Delta x$$

Express the work (in J) as an integral in terms of x (in m).

$$\int_0^{\square \; \boxed{16}} \left(\square \; \boxed{9.8(70.8 - 3.8x)} \right) dx$$

Evaluate the integral (in J). (Round your answer to the nearest integer.)

$\square$ $\boxed{6335}$ J

Skip to End to see it finally done right.

Variable Force(s)

Leaky Bucket $F = 10\,\text{Kg} + B(x)$

Rope : $\frac{.8\,\text{Kg}}{\text{m}}$

Bucket's Leakage rate : 48 Kg at start. $(0, 48)$

0 @ the end $(16, 0)$

mass of water :

$$B(x) = m(x - x_0) + B_0 = \frac{48 - 0}{0 - 16}(x - 0) + 48$$

$$= -3x + 48 \rightarrow \text{wt of water.}$$

$$\int_0^{16}$$

weight of rope : $(0, 16 \cdot .8) = (0, 12.8)$

$(16, 0) = (16, 0)$

16 meters of rope : $(16\,\text{m})\left(\frac{.8\,\text{Kg}}{\text{m}}\right) = 12.8$

$$R(x) = m(x - x_0) + R_0 = \frac{12.8}{-16}(x - 0) + 12.8$$

$$= -.8x + 12.8 = R(x)$$

x = distance from the bottom of the well.

$$\text{Force} = F = F(x) = (m_w + m_R)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)$$

$$F = ma$$

$$= \underbrace{(-3x + 48 + -.8x + 12.8)}_{\text{Kg}}\underbrace{(9.8)}_{\text{m/s}^2} = -37.24x + 595.84$$

-37.24*x + 595.84

Pull out the ugly 9.8 :

$$9.8(2.2x + 50.8)$$

$2.2x \to -3.8$, idiot. $50.8 \to 60.8$

Now, total work done in this futile exercise:

$$W = FD \approx \sum_{k=1}^{n} F(x_k)\Delta x \xrightarrow{n \to \infty} 9.8\int_0^{16} (-3.8x + 50.8)\,dx \approx 3198.720000$$

$50.8 \to 70.8$

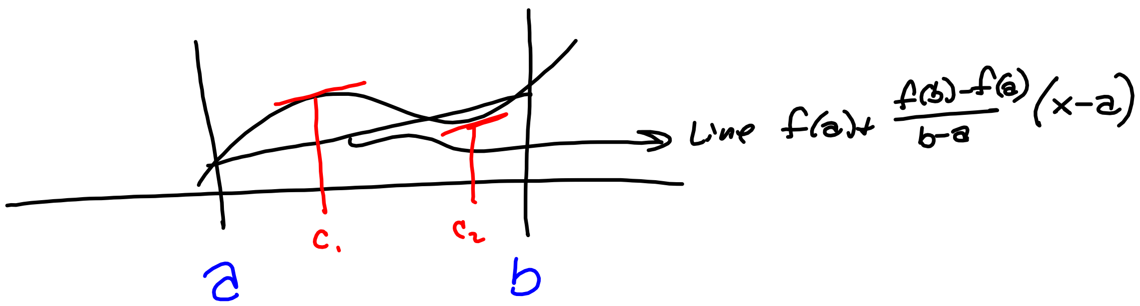
$\boxed{\text{Idiot left out the 10 Kg Bucket's Mass !}}$

$\boxed{\text{Idiot added 48 \& 12.8 \& got 50.8}}$

Recall Mean Value Theorem for Functions

f cnt^s on $[a,b]$, f difbl on (a,b) $\Longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$



"Mean Value" of a function

f cnt^s on $[a,b] \Longrightarrow \exists\, c \in (a,b) \ni f(c) = f_{avg}$ on $[a,b]$

$\nexists$ $\boxed{f_{avg} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx}$

6. 0/3 points SCalc9 5.5.011. [4934630]

Consider the following given function and given interval.

$f(x) = 16\sin(x) - 8\sin(2x), \quad [0, \pi]$

(a) Find the average value f_{ave} of f on the given interval.

$f_{ave} =$ $\frac{32}{\pi}$

(b) Find c in the given interval such that $f_{ave} = f(c)$. (Enter your answers as a comma-separated list. Round your answers to three decimal places.)

$c =$ 1.238, 2.808

(c) Sketch the graph of f and a rectangle whose base is the given interval and whose area is the same as the area under the graph of f.

(a) $\frac{1}{\pi}\int_0^{\pi} (16\sin(x) - 8\sin(2x))\,dx = \frac{32}{\pi}$ according to Symp.

(b) solve $16\sin(x) - 8\sin(2x) = \frac{32}{\pi}$

$\Rightarrow 16\sin(x) - 16\sin(x)\cos(x) = \frac{32}{\pi}$

Graphing Calc.

MAPLE

wolframalpha.com

Desmos