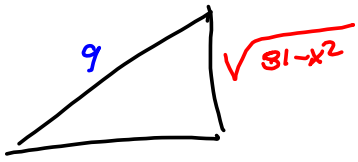
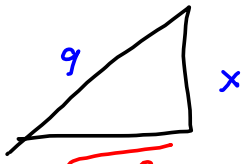
Say hey!

$$\int \frac{x^3}{\sqrt{81-x^2}}\,dx =$$



$x = 9\cos\theta$

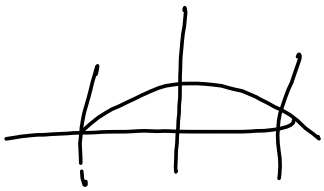


$x = 9\cos\theta$

$$\int \frac{729\sin^3\theta}{\sqrt{\cos^2\theta}}\,d\theta$$

$$= \int \frac{729\sin^3\theta\sin\theta}{\cos\theta^*}\,d\theta$$

$$*\text{b/c } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$



$$\int \frac{729(1-\cos^2\theta)\sin\theta}{\cos\theta}$$

$$x = 9\sin\theta$$

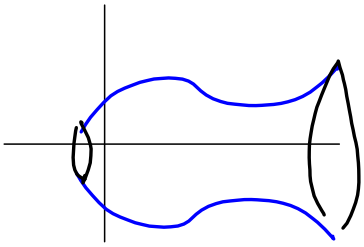
$$dx = 9\cos\theta\,d\theta$$

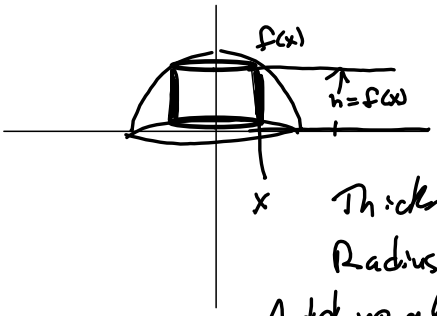
$$\int \frac{9\sin^3\theta}{9\cos\theta}\,9\cos\theta\,d\theta = \int \sin^3\theta\,d\theta \cdot 729 \quad :$$

$$= 729\int \sin^3\theta\,d\theta$$

§5.3 Volume by cylindrical shells

Rotate $f(x)$ about the x-axis.



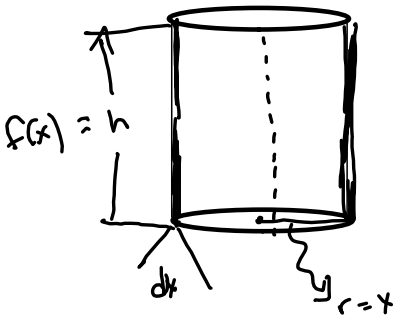


Rotate $f(x)$ about y-axis (Easiest!)

Thickness of shell is dx

Radius of shell is x

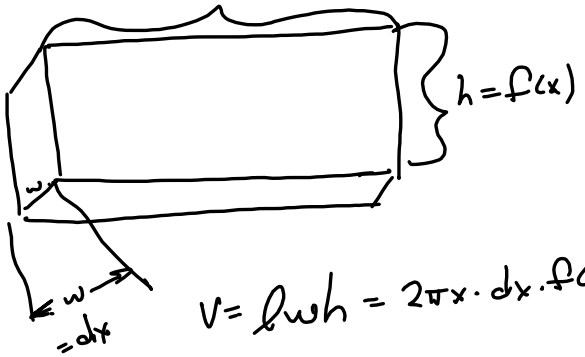
Add up all the shells



Volume of the soup can's side-wall:

Lay it out flat

l = circumference $= 2\pi r = 2\pi x$



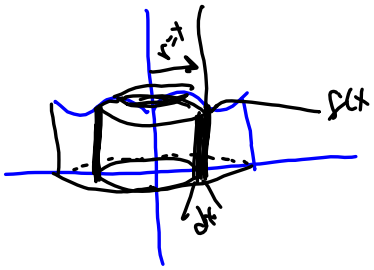
$V = lwh = 2\pi x \cdot dx \cdot f(x)$

$V = \sum$ boxes

$= \sum_{i=1}^{n} 2\pi \times f(x_i)\Delta x$

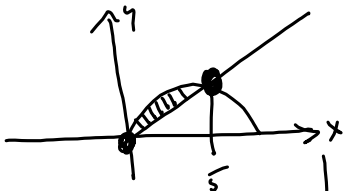
$n \to \infty$ $\quad 2\pi \int_0^b x f(x)\,dx$

Volume by cylindrical shells.



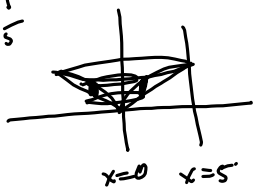
$y = 6x - x^2$, $y = x$, about y-axis

$V = 2\pi \int_a^b x f(x)\,dx$



$6x - x^2 =$
$-x^2 + 6x$
$= x(x-6)$
$-x^2 + 6x = x$
$-x^2 + 5x = 0$
$-x(x-5) = 0$

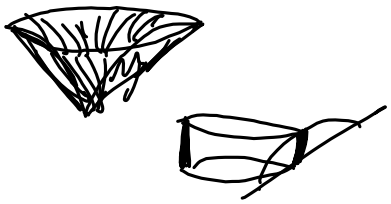
2π



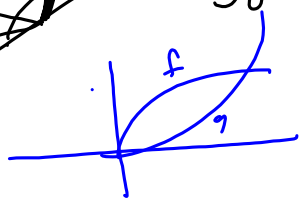
$2\pi \int_0^5 x\,(6x - x^2 - x)\,dx$

$= 2\pi \int_0^5 x\,(5x - x^2)\,dx$

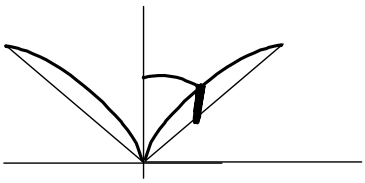
$= 2\pi \int_0^5 (5x^2 - x^3)\,dx$



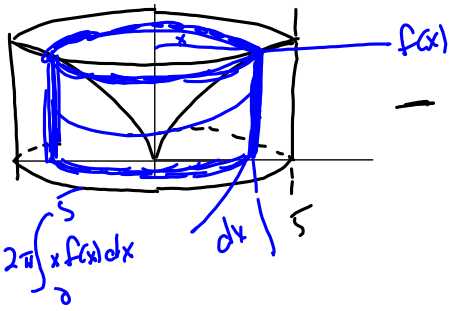
$\int_0^5$



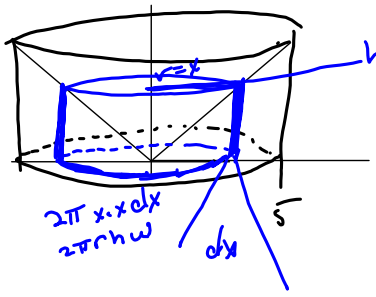
$\int (f - g)$



$2\pi \int_0^5 x\,(6x - x^2)\,dx - 2\pi \int_0^5 x\,dx$



$2\pi \int_0^5 x f(x)\,dx$



$2\pi \cdot x \cdot x\,dx$
$2\pi r h w$

$2\pi \int_0^5 x\,(6x - x^2)\,dx - \int_0^5 x \cdot x\,dx$

$2\pi \int x\,(\text{upper} - \text{lower})\,dx$