$f := x \to \exp(-x^2)$

$$f := x \mapsto e^{-x^2} \tag{1}$$

$\int_{-2}^{6} \exp(-x^2)\, dx$

$$\frac{\operatorname{erf}(2)\sqrt{\pi}}{2} + \frac{\operatorname{erf}(6)\sqrt{\pi}}{2} \tag{2}$$

evalf (%)

$$1.768308316 \tag{3}$$

$\frac{4}{5} \cdot \sum_{k=1}^{100} f\left(-2 + \frac{k \cdot 4}{5}\right)$

$$\frac{4\,e^{-\frac{196}{25}}}{5} + \frac{4\,e^{-\frac{324}{25}}}{5} + \frac{4\,e^{-\frac{484}{25}}}{5} + \frac{4\,e^{-\frac{676}{25}}}{5} + \frac{8\,e^{-\frac{36}{25}}}{5} + \frac{8\,e^{-\frac{4}{25}}}{5} + \frac{4\,e^{-36}}{5} + \frac{4\,e^{-4}}{5}$$
$$+ \frac{4\,e^{-\frac{1156}{25}}}{5} + \frac{4\,e^{-\frac{1444}{25}}}{5} + \frac{4\,e^{-\frac{1764}{25}}}{5} + \frac{4\,e^{-\frac{2116}{25}}}{5} + \frac{4\,e^{-100}}{5} + \frac{4\,e^{-\frac{2916}{25}}}{5}$$
$$+ \frac{4\,e^{-\frac{3364}{25}}}{5} + \frac{4\,e^{-\frac{3844}{25}}}{5} + \frac{4\,e^{-\frac{4356}{25}}}{5} + \frac{4\,e^{-196}}{5} + \frac{4\,e^{-\frac{5476}{25}}}{5} + \frac{4\,e^{-\frac{6084}{25}}}{5}$$
$$+ \frac{4\,e^{-\frac{6724}{25}}}{5} + \frac{4\,e^{-\frac{7396}{25}}}{5} + \frac{4\,e^{-324}}{5} + \frac{4\,e^{-\frac{8836}{25}}}{5} + \frac{4\,e^{-\frac{9604}{25}}}{5} + \frac{4\,e^{-\frac{10404}{25}}}{5}$$
$$+ \frac{4\,e^{-\frac{11236}{25}}}{5} + \frac{4\,e^{-484}}{5} + \frac{4\,e^{-\frac{12996}{25}}}{5} + \frac{4\,e^{-\frac{13924}{25}}}{5} + \frac{4\,e^{-\frac{14884}{25}}}{5} + \frac{4\,e^{-\frac{15876}{25}}}{5}$$
$$+ \frac{4\,e^{-676}}{5} + \frac{4\,e^{-\frac{17956}{25}}}{5} + \frac{4\,e^{-\frac{19044}{25}}}{5} + \frac{4\,e^{-\frac{20164}{25}}}{5} + \frac{4\,e^{-\frac{21316}{25}}}{5} + \frac{4\,e^{-900}}{5}$$
$$+ \frac{4\,e^{-\frac{23716}{25}}}{5} + \frac{4\,e^{-\frac{24964}{25}}}{5} + \frac{4\,e^{-\frac{26244}{25}}}{5} + \frac{4\,e^{-\frac{27556}{25}}}{5} + \frac{4\,e^{-1156}}{5} + \frac{4\,e^{-\frac{30276}{25}}}{5}$$
$$+ \frac{4\,e^{-\frac{31684}{25}}}{5} + \frac{4\,e^{-\frac{33124}{25}}}{5} + \frac{4\,e^{-\frac{34596}{25}}}{5} + \frac{4\,e^{-1444}}{5} + \frac{4\,e^{-\frac{37636}{25}}}{5} + \frac{4\,e^{-\frac{39204}{25}}}{5}$$
$$+ \frac{4\,e^{-\frac{40804}{25}}}{5} + \frac{4\,e^{-\frac{42436}{25}}}{5} + \frac{4\,e^{-1764}}{5} + \frac{4\,e^{-\frac{45796}{25}}}{5} + \frac{4\,e^{-\frac{47524}{25}}}{5} + \frac{4\,e^{-\frac{49284}{25}}}{5}$$
$$+ \frac{4\,e^{-\frac{51076}{25}}}{5} + \frac{4\,e^{-2116}}{5} + \frac{4\,e^{-\frac{54756}{25}}}{5} + \frac{4\,e^{-\frac{56644}{25}}}{5} + \frac{4\,e^{-\frac{58564}{25}}}{5} + \frac{4\,e^{-\frac{60516}{25}}}{5} \tag{4}$$

$$+\frac{4\,e^{-2500}}{5}+\frac{4\,e^{-\frac{64516}{25}}}{5}+\frac{4\,e^{-\frac{66564}{25}}}{5}+\frac{4\,e^{-\frac{68644}{25}}}{5}+\frac{4\,e^{-\frac{70756}{25}}}{5}+\frac{4\,e^{-2916}}{5}$$
$$+\frac{4\,e^{-\frac{75076}{25}}}{5}+\frac{4\,e^{-\frac{77284}{25}}}{5}+\frac{4\,e^{-\frac{79524}{25}}}{5}+\frac{4\,e^{-\frac{81796}{25}}}{5}+\frac{4\,e^{-3364}}{5}+\frac{4\,e^{-\frac{86436}{25}}}{5}$$
$$+\frac{4\,e^{-\frac{88804}{25}}}{5}+\frac{4\,e^{-\frac{91204}{25}}}{5}+\frac{4\,e^{-\frac{93636}{25}}}{5}+\frac{4\,e^{-3844}}{5}+\frac{4\,e^{-\frac{98596}{25}}}{5}+\frac{4\,e^{-\frac{101124}{25}}}{5}$$
$$+\frac{4\,e^{-\frac{103684}{25}}}{5}+\frac{4\,e^{-\frac{106276}{25}}}{5}+\frac{4\,e^{-4356}}{5}+\frac{4\,e^{-\frac{111556}{25}}}{5}+\frac{4\,e^{-\frac{114244}{25}}}{5}+\frac{4\,e^{-\frac{116964}{25}}}{5}$$
$$+\frac{4\,e^{-\frac{119716}{25}}}{5}+\frac{4\,e^{-4900}}{5}+\frac{4\,e^{-\frac{125316}{25}}}{5}+\frac{4\,e^{-\frac{128164}{25}}}{5}+\frac{4\,e^{-\frac{131044}{25}}}{5}+\frac{4\,e^{-\frac{133956}{25}}}{5}$$
$$+\frac{4\,e^{-5476}}{5}+\frac{4\,e^{-\frac{139876}{25}}}{5}+\frac{4\,e^{-\frac{142884}{25}}}{5}+\frac{4\,e^{-\frac{145924}{25}}}{5}+\frac{4\,e^{-\frac{148996}{25}}}{5}+\frac{4\,e^{-6084}}{5}$$
$$2\,e^{-\frac{36}{25}}+2\,e^{-\frac{4}{25}}+e^{-4}+e^{-\frac{196}{25}}+e^{-\frac{324}{25}}+e^{-\frac{484}{25}}+e^{-\frac{676}{25}}+e^{-36} \quad \textbf{(5)}$$

evalf(%)

$$1.757483807 \quad \textbf{(6)}$$

$\Delta x := \frac{4}{5}$

$$\Delta x := \frac{4}{5} \quad \textbf{(7)}$$

$a := -2$

$$a := -2 \quad \textbf{(8)}$$

$b := 6$

$$b := 6 \quad \textbf{(9)}$$

$$\frac{\Delta x}{3}\cdot\left(f(a)+4\cdot f(a+\Delta x)+2\cdot f(a+2\cdot\Delta x)+4\cdot f(a+3\cdot\Delta x)+2\cdot f(a+4\,\Delta x)+4\cdot f(a+5\cdot\Delta x)\right.$$
$$\left.+2\cdot f(a+6\cdot\Delta x)+4\cdot f(a+7\cdot\Delta x)+2\cdot f(a+8\cdot\Delta x)+4\cdot f(a+9\cdot\Delta x)+f(a+10\cdot\Delta x)\right)$$
$$\frac{4\,e^{-4}}{3}+\frac{8\,e^{-\frac{36}{25}}}{5}+\frac{8\,e^{-\frac{4}{25}}}{5}+\frac{8\,e^{-\frac{196}{25}}}{15}+\frac{16\,e^{-\frac{324}{25}}}{15}+\frac{8\,e^{-\frac{484}{25}}}{15}+\frac{16\,e^{-\frac{676}{25}}}{15}+\frac{4\,e^{-36}}{15} \quad \textbf{(10)}$$

evalf(%)

$$1.767147796 \quad \textbf{(11)}$$

$a + 10\cdot\Delta x$

$$6 \quad \textbf{(12)}$$

ApproximateInt(exp(-x^2), -2 .. 6, '*partition*' = 10, '*method*' = *simpson*, '*partitiontype*' = *normal*, '*output*' = '*value*', '*boxoptions*' = ['*filled*' = ['*transparency*' = .5]]);

$$\frac{4\,e^{-4}}{3}+\frac{8\,e^{-\frac{36}{25}}}{5}+\frac{8\,e^{-\frac{4}{25}}}{5}+\frac{8\,e^{-\frac{196}{25}}}{15}+\frac{16\,e^{-\frac{324}{25}}}{15}+\frac{8\,e^{-\frac{484}{25}}}{15}+\frac{16\,e^{-\frac{676}{25}}}{15}+\frac{4\,e^{-36}}{15} \tag{13}$$

evalf(%)

$$1.767147796 \tag{14}$$

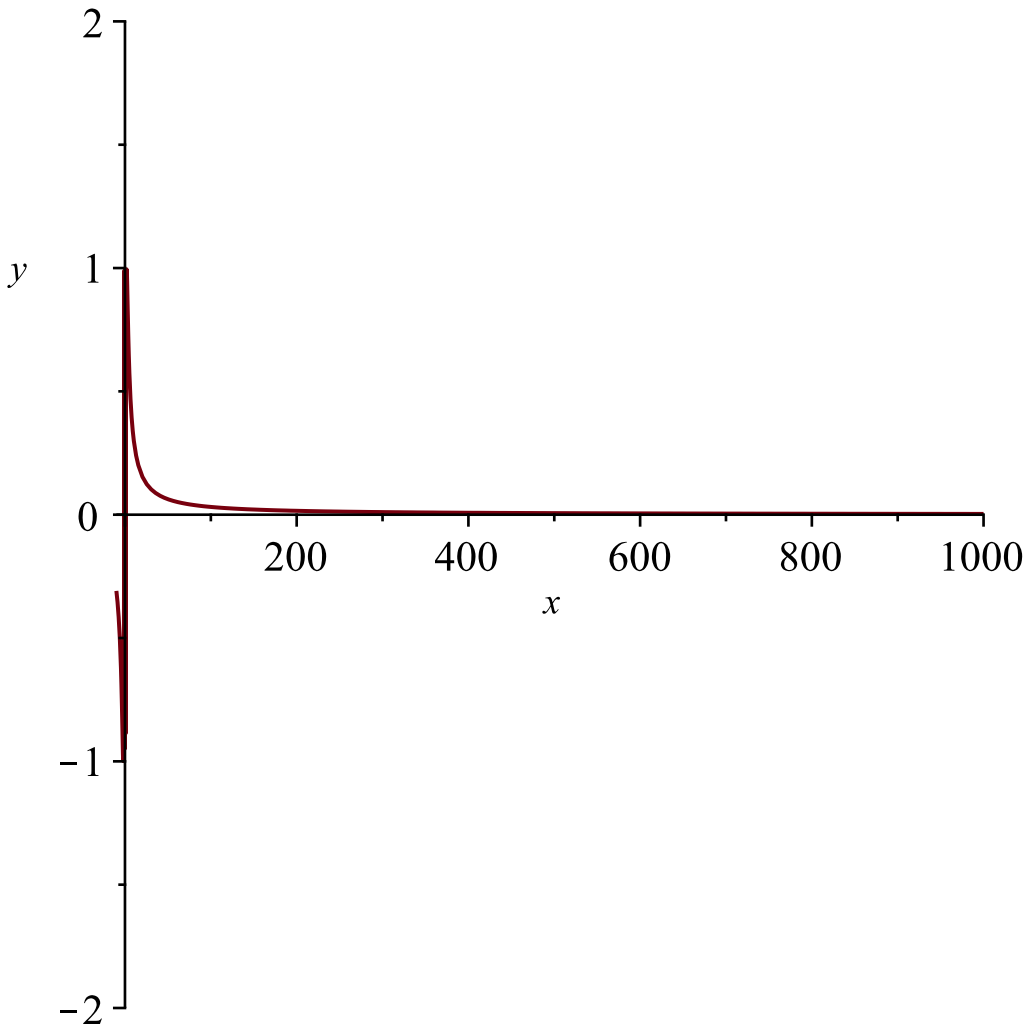
$$\int_{-2}^{6}\exp(-x^2)\,dx$$

$$\frac{\mathrm{erf}(2)\sqrt{\pi}}{2}+\frac{\mathrm{erf}(6)\sqrt{\pi}}{2} \tag{15}$$

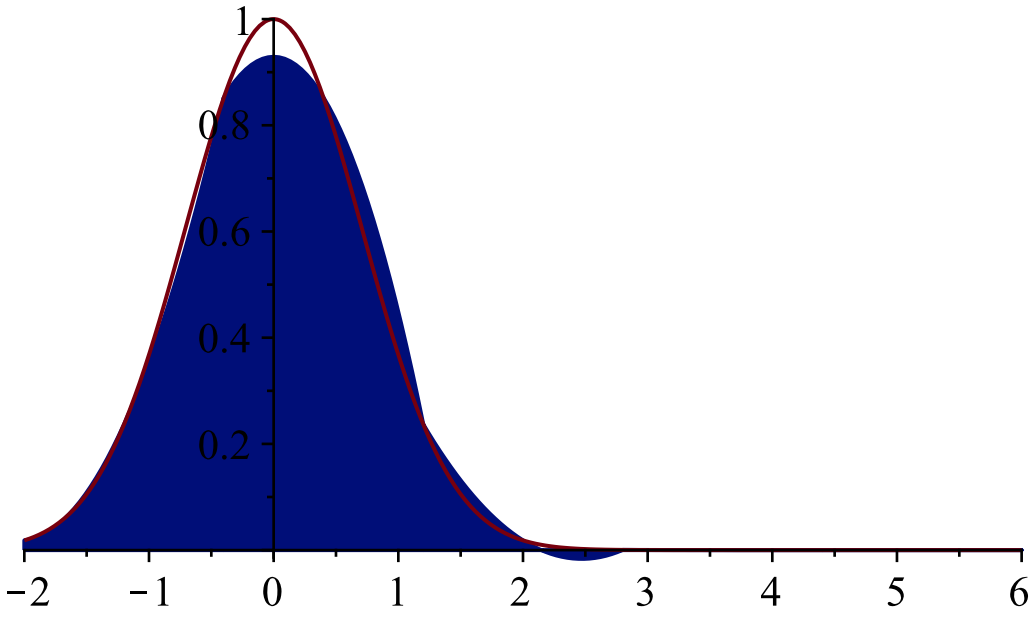
evalf(%)

$$1.768308316 \tag{16}$$

```
with(Student[Calculus1]):
with(plots):
plot(sin(Pi/x), x=-10..1000, y=-2..2)
```




```
ApproximateInt(exp(-x^2), -2 .. 6, 'partition' = 10, 'method' = simpson, 'partitiontype' = normal,
    'output' = 'plot', 'boxoptions' = ['filled' = ['transparency' = .5]]);
```



An approximation of $\int_{-2}^{6} f(x)\,dx$ using Simpson's rule,

where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 1.767147796. Number of subintervals used: 5.

```
?ApproximateInt
```

```
f(x)
```

$$e^{-x^2} \quad \textbf{(17)}$$

```
fp := D(f)
```

$$fp := x \mapsto -2\,x\,e^{-x^2} \quad \textbf{(18)}$$

```
fp2 := D(fp)
```

$$fp2 := x \mapsto -2\,e^{-x^2} + 4\,x^2\,e^{-x^2} \quad \textbf{(19)}$$

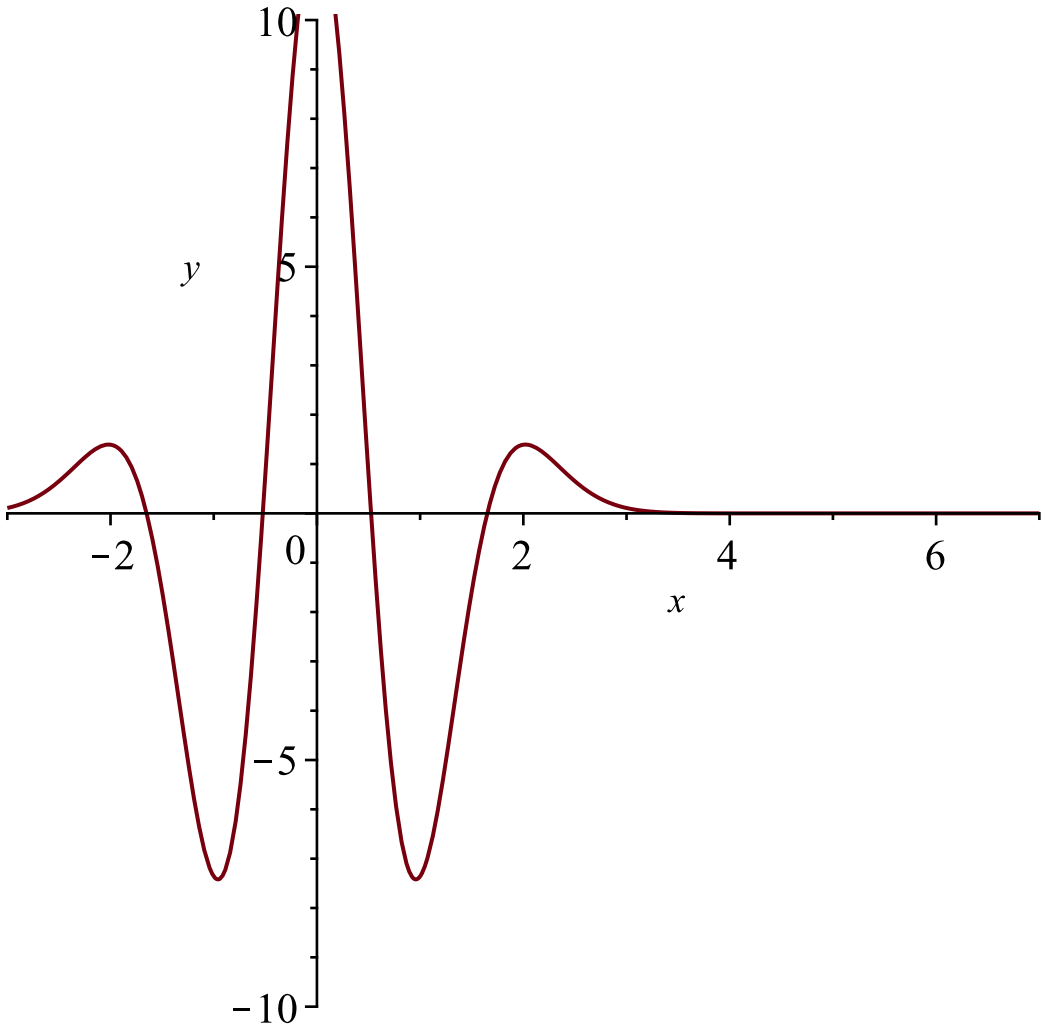
```
fp3 := D(fp2)
```

$$fp3 := x \mapsto 12\,x\,e^{-x^2} - 8\,x^3\,e^{-x^2} \quad \textbf{(20)}$$

```
fp4 := D(fp3)
```

$$fp4 := x \mapsto 12\,e^{-x^2} - 48\,x^2\,e^{-x^2} + 16\,x^4\,e^{-x^2} \quad \textbf{(21)}$$

```
plot(fp4(x), x = -3..7, y = -10..10)
```



$fp5 := \mathrm{D}(fp4)$

$$fp5 := x \mapsto -120\,x\,e^{-x^2} + 160\,x^3\,e^{-x^2} - 32\,x^5\,e^{-x^2} \quad \textbf{(22)}$$

$solve(fp5(x) = 0)$

$$0,\ \frac{\sqrt{10-2\sqrt{10}}}{2},\ -\frac{\sqrt{10-2\sqrt{10}}}{2},\ \frac{\sqrt{10+2\sqrt{10}}}{2},\ -\frac{\sqrt{10+2\sqrt{10}}}{2} \quad \textbf{(23)}$$

$evalf(\%)$

$$0.,\ 0.9585724645,\ -0.9585724645,\ 2.020182870,\ -2.020182870 \quad \textbf{(24)}$$

$fp4(-0.9585724645)$

$$-7.419481177 \quad \textbf{(25)}$$

$fp4(2.020182870)$

$$1.394907051 \quad \textbf{(26)}$$

$fp4(-2.020182870)$

$$1.394907051 \quad \textbf{(27)}$$

$fp4(-2)$

$$76\,e^{-4} \quad \textbf{(28)}$$

$evalf(\%)$

$$1.391988556 \quad \textbf{(29)}$$

$fp4(6)$

$$19020\ e^{-36} \tag{30}$$

$evalf(\%)$

$$4.411732423\ 10^{-12} \tag{31}$$