$f := x \to \exp(-x^2)$

$$f := x \mapsto e^{-x^2} \tag{1}$$

$\int_{-2}^{6} \exp(-x^2)\, dx$

$$\frac{\operatorname{erf}(2)\sqrt{\pi}}{2} + \frac{\operatorname{erf}(6)\sqrt{\pi}}{2} \tag{2}$$

$evalf(\%)$

$$1.768308316 \tag{3}$$

$right\_sum := \frac{8}{n} \cdot \sum_{k=1}^{n} f\left(-2 + \frac{k \cdot 4}{5}\right)$

$$right\_sum := \frac{4\left(\sum_{k=1}^{n} e^{-\left(-2+\frac{4k}{5}\right)^2}\right)}{5} \tag{4}$$

$left\_sum := \frac{8}{n} \cdot \sum_{k=0}^{n-1} f\left(-2 + \frac{k \cdot 4}{5}\right)$

$$left\_sum := \frac{8\left(\sum_{k=0}^{n-1} e^{-\left(-2+\frac{4k}{5}\right)^2}\right)}{n} \tag{5}$$

$left\_sum - right\_sum$

$$\frac{8\left(\sum_{k=0}^{n-1} e^{-\left(-2+\frac{4k}{5}\right)^2}\right)}{n} - \frac{4\left(\sum_{k=1}^{n} e^{-\left(-2+\frac{4k}{5}\right)^2}\right)}{5} \tag{6}$$

$simplify(\%)$

$$\left(\sum_{k=0}^{n-1} e^{-\frac{4(-5+2k)^2}{25}}\right) - \frac{4\left(\sum_{k=1}^{n} e^{-\frac{4(-5+2k)^2}{25}}\right)}{5} \tag{7}$$

$\Delta x := \frac{4}{5}$

$$\Delta x := \frac{4}{5} \tag{8}$$

$a := -2$

$$a := -2 \tag{9}$$

$b := 6$

$$b := 6 \tag{10}$$

$$\frac{\Delta x}{3}\cdot\left(f(a)+4\cdot f(a+\Delta x)+2\cdot f(a+2\cdot\Delta x)+4\cdot f(a+3\cdot\Delta x)+2\cdot f(a+4\,\Delta x)+4\cdot f(a+5\cdot\Delta x)\right.$$
$$\left.+2\cdot f(a+6\cdot\Delta x)+4\cdot f(a+7\cdot\Delta x)+2\cdot f(a+8\cdot\Delta x)+4\cdot f(a+9\cdot\Delta x)+f(a+10\cdot\Delta x)\right)$$

$$\frac{4\,e^{-4}}{3}+\frac{8\,e^{-\frac{36}{25}}}{5}+\frac{8\,e^{-\frac{4}{25}}}{5}+\frac{8\,e^{-\frac{196}{25}}}{15}+\frac{16\,e^{-\frac{324}{25}}}{15}+\frac{8\,e^{-\frac{484}{25}}}{15}+\frac{16\,e^{-\frac{676}{25}}}{15}+\frac{4\,e^{-36}}{15} \qquad \textbf{(11)}$$

evalf(%)

$$1.767147796 \qquad \textbf{(12)}$$

$a+10\cdot\Delta x$

$$6 \qquad \textbf{(13)}$$

ApproximateInt(exp(-*x*^2), -2 .. 6, '*partition*' = 10, '*method*' = *simpson*, '*partitiontype*' = *normal*, '*output*' = '*value*', '*boxoptions*' = ['*filled*' = ['*transparency*'= .5]]);

$$\frac{4\,e^{-4}}{3}+\frac{8\,e^{-\frac{36}{25}}}{5}+\frac{8\,e^{-\frac{4}{25}}}{5}+\frac{8\,e^{-\frac{196}{25}}}{15}+\frac{16\,e^{-\frac{324}{25}}}{15}+\frac{8\,e^{-\frac{484}{25}}}{15}+\frac{16\,e^{-\frac{676}{25}}}{15}+\frac{4\,e^{-36}}{15} \qquad \textbf{(14)}$$

evalf(%)

$$1.767147796 \qquad \textbf{(15)}$$

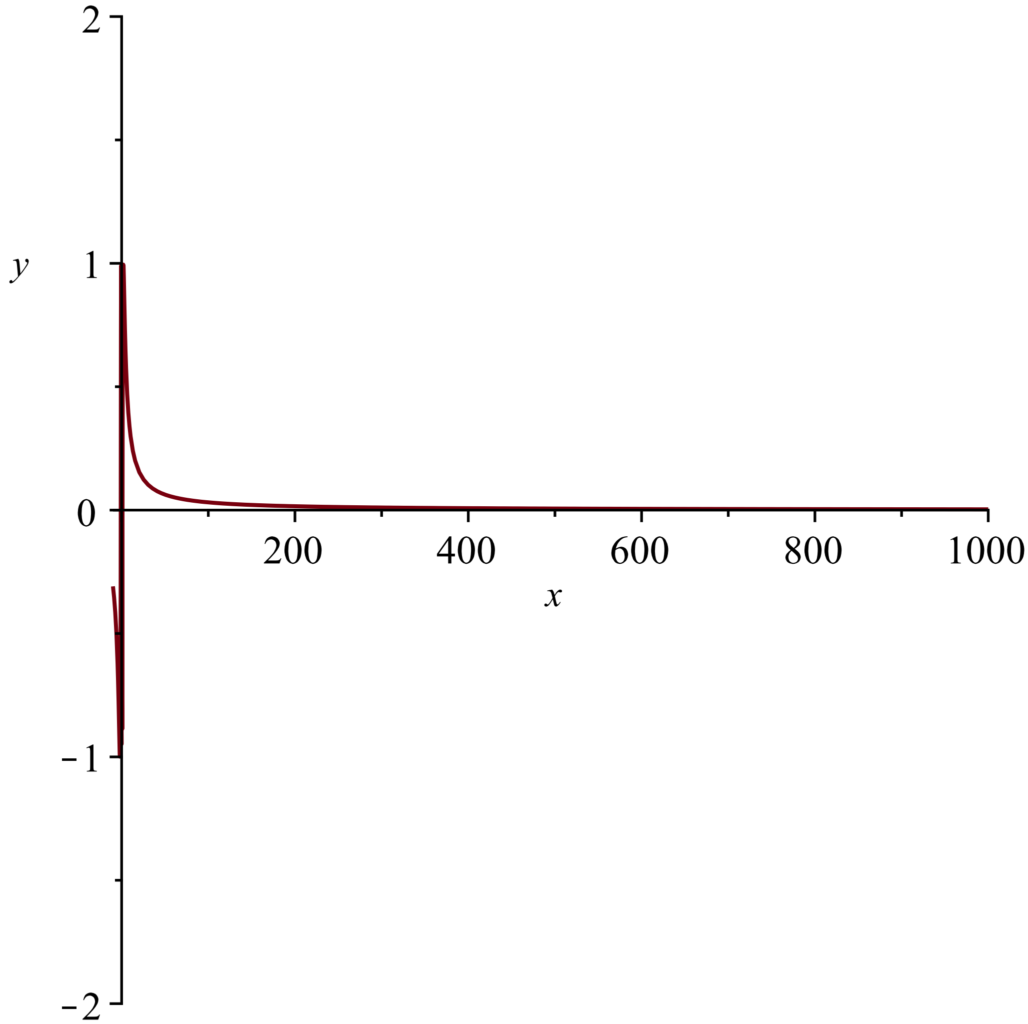
$$\int_{-2}^{6}\exp(-x^2)\,dx$$

$$\frac{\operatorname{erf}(2)\sqrt{\pi}}{2}+\frac{\operatorname{erf}(6)\sqrt{\pi}}{2} \qquad \textbf{(16)}$$

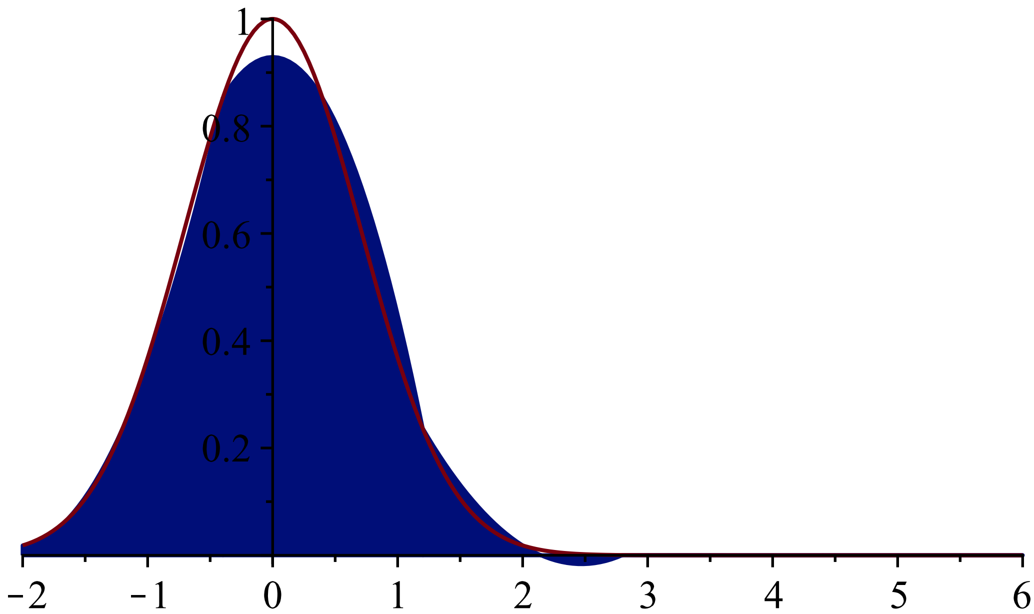
evalf(%)

$$1.768308316 \qquad \textbf{(17)}$$

with(*Student*[*Calculus1*]) :
with(*plots*) :
$plot\left(\sin\left(\frac{\text{Pi}}{x}\right), x=-10\,..1000, y=-2\,..2\right)$



ApproximateInt(exp(−*x*^2), −2 .. 6, '*partition*' = 10, '*method*' = *simpson*, '*partitiontype*' = *normal*, '*output*' = '*plot*', '*boxoptions*' = ['*filled*' = ['*transparency*'= .5]]);



An approximation of $\int_{-2}^{6} f(x)\,dx$ using Simpson's rule, where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 1.767147796. Number of subintervals used: 5.

?*ApproximateInt*

$f(x)$

$$e^{-x^2} \tag{18}$$

$fp := D(f)$

$$fp := x \mapsto -2\,x\,e^{-x^2} \tag{19}$$

$fp2 := D(fp)$

$$fp2 := x \mapsto -2\,e^{-x^2} + 4\,x^2\,e^{-x^2} \tag{20}$$

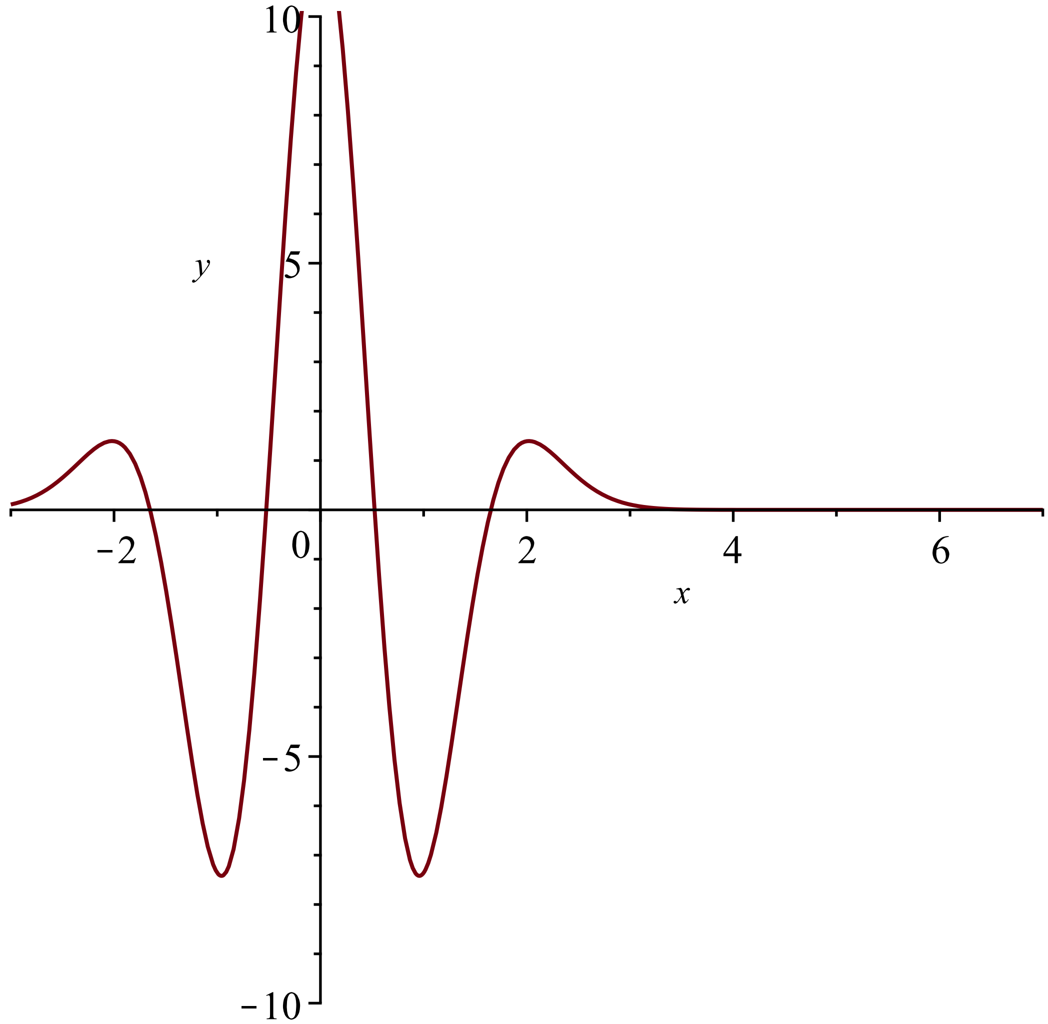
$fp3 := D(fp2)$

$$fp3 := x \mapsto 12\,x\,e^{-x^2} - 8\,x^3\,e^{-x^2} \tag{21}$$

$fp4 := D(fp3)$

$$fp4 := x \mapsto 12\,e^{-x^2} - 48\,x^2\,e^{-x^2} + 16\,x^4\,e^{-x^2} \tag{22}$$

$plot(fp4(x), x=-3..7, y=-10..10)$



$fp5 := D(fp4)$

$$fp5 := x \mapsto -120\, x\, e^{-x^2} + 160\, x^3\, e^{-x^2} - 32\, x^5\, e^{-x^2} \quad \textbf{(23)}$$

$solve(fp5(x) = 0)$

$$0, \frac{\sqrt{10 - 2\sqrt{10}}}{2}, -\frac{\sqrt{10 - 2\sqrt{10}}}{2}, \frac{\sqrt{10 + 2\sqrt{10}}}{2}, -\frac{\sqrt{10 + 2\sqrt{10}}}{2} \quad \textbf{(24)}$$

$evalf(\%)$

$$0., 0.9585724645, -0.9585724645, 2.020182870, -2.020182870 \quad \textbf{(25)}$$

$fp4(-0.9585724645)$

$$-7.419481177 \quad \textbf{(26)}$$

$fp4(2.020182870)$

$$1.394907051 \quad \textbf{(27)}$$

$fp4(-2.020182870)$

$$1.394907051 \quad \textbf{(28)}$$

$fp4(-2)$

$$76\, e^{-4} \quad \textbf{(29)}$$

$evalf(\%)$

$$1.391988556 \quad \textbf{(30)}$$

$fp4(6)$

$$19020\, e^{-36} \tag{31}$$

$evalf(\%)$

$$4.411732423\ 10^{-12} \tag{32}$$

$fp4(0)$

$$12 \tag{33}$$