MAT 202 | Test 2, Spring, 2020 | Name ________________

80 Points | Covers Chapter 7

Do all your work and put all your answers WITH your work, CIRCLED, on the white paper provided. All I want on this sheet is your NAME! Spend no more than 2 minutes on any single problem on your first pass through the test. If you don't finish a problem in 2 or 3 minutes, start a fresh sheet of paper for the next problem, and so on.

Formatting should be the same as homework, only you don't need to re-state the question, because the question's attached to your test, and you're under a time control.

1. (15 pts) Evaluate $\int x\ln\left(\sqrt[5]{x}\right)dx$

2. (15 pts) Evaluate $\int x\sin\left(x\right)dx$

3. Evaluate $\int_2^{2\sqrt{2}} \frac{x}{\sqrt{x^2-4}}dx$ in two ways:

   a. (10 pts) By *u*-substitution.

   b. (10 pts) By trigonometric substitution.

4. (10 pts) Evaluate $\int \frac{x-23}{x^2-x-6}dx$

5. (10 pts) Evaluate $\int \sin\left(5x\right)\cos\left(3x\right)\,dx$

6. (10 pts) Evaluate $\int \sin^4 x\cos^3 x\,dx$

Work up to 15 points' worth of Bonus, below:

Bonus 1: (10 pts) Evaluate $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \sqrt{1-\sin\left(x\right)}\,dx$. This relates to the question asked in class on Wednesday.

Bonus 2: (5 pts) Evaluate $\int_e^{\infty} \frac{dx}{x\ln\left(x\right)^3}$

Bonus 3: (5 pts) Evaluate $\int_{-1}^{2\sqrt{2}-3} \frac{x+3}{\sqrt{x^2+6x+5}}dx$

Bonus 4: (5 pts) Evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$

MAT 202 Test 2, Spring, 2020 Name__________________________

20 Points Take-Home

I'll give you all of next week to access technology

1. (5 pts) Evaluate $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \sqrt{1-\sin(x)}\,dx$. This relates to the question asked in class on Wednesday.

2. (5 pts) Use Simpson's rule, with $n = 10$ to approximate $\int_{-2}^{6} e^{-x^2}\,dx$. Give an upper bound on the error for this method on this integral.

3. (5 pts) Use a right-endpoint Riemann sums to approximate $\int_{-2}^{6} e^{-x^2}\,dx$, accurate to 4 decimal places.

4. (5 pts) Make 2 arguments for the convergence of the improper integral: $\int_{1}^{\infty} \frac{x^2 dx}{\sqrt[4]{x^{13}+x^2+1}}$.

   a. Make a "general-idea" argument, with a rough comparison to an appropriate *p*-function.

   b. Make a more formal argument, with a direct comparison to the appropriate *p*-function.

Bonus Here are two tricky direct-comparison problems that requre some more advanced analysis. I want your intuition to be strong on these, but I also want you to be at least passingly familiar with more advanced "How big IS it?" techniques. In the sequel (Infinite series in Chapter 11), all you will need is a "limit comparison" and everything acts the way you want it to, if you can just generally see, for instance, that $x^{\frac{8}{3}} - 2x^2 + 11x^{\frac{1}{3}}$ is eventually dominated by the $x^{\frac{8}{3}}$.

Bonus 1: (5 pts) Make a direct comparison to show that $\int_{2}^{\infty} \frac{dx}{\sqrt{x^3}-\sqrt{x}}$ converges. That is, show, explicitly, that the integrand is less than or equal to a *p*-function whose improper integral is known to converge. This is tricky, because it's hard to make the $-\sqrt{x}$ go away without making the quotient smaller.

Bonus 2: (5 pts) Make a direct comparison to show that $\int_{2}^{\infty} \frac{dx}{\sqrt{x}+x}$ diverges, that is, show, explicitly, that the integrand is greater than or equal to a *p*-function whose improper integral is known to diverge. This is tricky, because it's hard to make the $\sqrt{x}$ go away without making the quotient bigger.

And yes, I really wanted the following to be in your hands while you took the test. I needed to make sure everyone had both pages while they took the test.

$$\sin A \cos B = \tfrac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)]$$