MAT 202
80 Points

Test 2, Spring, 2020
Covers Chapter 7

Name KEY

Do all your work and put all your answers WITH your work, CIRCLED, on the white paper provided. All I want on this sheet is your NAME! Spend no more than 2 minutes on any single problem on your first pass through the test. If you don't finish a problem in 2 or 3 minutes, start a fresh sheet of paper for the next problem, and so on.

Formatting should be the same as homework, only you don't need to re-state the question, because the question's attached to your test, and you're under a time control.

1. (15 pts) Evaluate $\int x\ln\left(\sqrt[5]{x}\right)dx$

2. (15 pts) Evaluate $\int x\sin(x)\,dx$

3. Evaluate $\int_{2}^{2\sqrt{2}} \frac{x}{\sqrt{x^2-4}}dx$ in two ways:

   a. (10 pts) By u-substitution.

   b. (10 pts) By trigonometric substitution.

4. (10 pts) Evaluate $\int \frac{x-23}{x^2-x-6}dx$

5. (10 pts) Evaluate $\int \sin(5x)\cos(3x)\,dx$

6. (10 pts) Evaluate $\int \sin^4 x\cos^3 x\,dx$

Work up to 15 points' worth of Bonus, below:

Bonus 1: (10 pts) Evaluate $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \sqrt{1-\sin(x)}\,dx$. This relates to the question asked in class on Wednesday.

Bonus 2: (5 pts) Evaluate $\int_{e}^{\infty} \frac{dx}{x\ln(x)^3}$

Bonus 3: (5 pts) Evaluate $\int_{-+1}^{2\sqrt{2}-3} \frac{x+3}{\sqrt{x^2+6x+5}}dx$

Bonus 4: (5 pts) Evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$

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(1) (15 pts) $\int x \ln \sqrt[5]{x}\, dx$

$u = \ln(x^{\frac{1}{5}}) = \frac{1}{5}\ln x$

$du = \frac{1}{5} \cdot \frac{1}{x}\, dx$

$dv = x\, dx$

$v = \frac{1}{2}x^2$

$= uv - \int v\, du = \frac{1}{10}x^2 \ln x$

$- \int \left(\frac{1}{2}x^2\right)\left(\frac{1}{5x}\, dx\right)$

$= \frac{1}{10}x^2 \ln x - \frac{1}{10}\int x\, dx = \boxed{\frac{1}{10}x^2 \ln x - \frac{1}{20}x^2 + C}$

(2) (15 pts) $\int x \sin x\, dx =$

$u = x \quad dv = \sin x\, dx$

$du = dx \quad v = -\cos x$

$= uv - \int v\, du = -x\cos x + \int \cos x\, dx$

$= \boxed{-x\cos x + \sin x + C}$

(3) (2) (10 pts) $\int_2^{2\sqrt{2}} \frac{x}{\sqrt{x^2-4}}\, dx =$

$u = x^2-4$

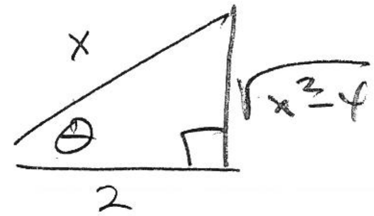
$du = 2x\, dx$

$= \frac{1}{2}\int_2^{2\sqrt{2}} (x^2-4)^{-\frac{1}{2}}(2x\, dx) = \frac{1}{2} \cdot 2(x^2-4)^{\frac{1}{2}}\Big]_2^{2\sqrt{2}}$

$= \left((2\sqrt{2})^2-4\right)^{\frac{1}{2}} - (2^2-4)^{\frac{1}{2}} = (8-4)^{\frac{1}{2}} = 4^{\frac{1}{2}} = \boxed{2}$

3b 10 pts

$$\int_2^{2\sqrt{2}} \frac{x}{\sqrt{x^2-4}}\,dx$$



Let $\frac{x}{2} = \sec\theta \longrightarrow x = 2\sec\theta$

$\longrightarrow dx = 2\sec\theta\tan\theta\, d\theta$

$x=2 \longrightarrow 2\sec\theta = 2 \Longrightarrow \sec\theta = 1 \Longrightarrow \theta = 0$

$x = 2\sqrt{2} \longrightarrow 2\sec\theta = 2\sqrt{2} \longrightarrow \sec\theta = \sqrt{2}$

$\longrightarrow \theta = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \frac{2\sec\theta \cdot 2\sec\theta\tan\theta\, d\theta}{2\tan\theta}$$

$$= 2\int_0^{\frac{\pi}{4}} \sec^2\theta\, d\theta = 2\tan\theta\Big]_0^{\frac{\pi}{4}} = 2(1) - 2(0) = \boxed{2}$$

4 10 pts $\displaystyle\int \frac{x-23}{x^2-x-6}\,dx$

M1 Partial Fractions

M2 Manipulate expression

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#4 cnt'd

M1 $\dfrac{x-23}{x^2-x-6} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$

$\Rightarrow x-23 = A(x+2) + B(x-3)$

$x=-2: \quad -2-23 = -25 = -5B \Rightarrow B=5$

$x=3: \quad 3-23 = -20 = 5A \Rightarrow A=-4$

$5\displaystyle\int \frac{dx}{x+2} - 4\int \frac{dx}{x-3} = \boxed{5\ln|x+2| - 4\ln|x-3| + C}$

M2 $u = x^2-x-6 \Rightarrow du = 2x-1$

$\displaystyle\int \frac{x-23}{x^2-x-6}dx = \frac{1}{2}\int \frac{2x-46}{x^2-x-6}dx = \boxed{\text{FORMULA 20}}$

$= \displaystyle\frac{1}{2}\int \frac{2x-1}{x^2-x-6}dx - \frac{1}{2}\int \frac{45\,dx}{x^2-x-6}$

$= \displaystyle\frac{1}{2}\int \frac{du}{u} - \frac{45}{2}\int \frac{dx}{x^2-x-6} = \frac{1}{2}\ln|x^2-x-6|$

$a^2 = \frac{25}{4} \Rightarrow a = \frac{5}{2}$

$\Rightarrow x^2-x-6 = x^2-x+\left(\frac{1}{2}\right)^2 - \frac{1}{4} - \frac{24}{4} = \left(x-\frac{1}{2}\right)^2 - \frac{25}{4}$

$-\displaystyle\frac{45}{2}\int \frac{du}{u^2-\frac{25}{4}}$

Let $u = \left(x-\frac{1}{2}\right) \Rightarrow du = dx \Rightarrow$ #20

$\frac{1}{2}\ln|x^2-x-6| - \frac{45}{2}\left[\frac{1}{2\left(\frac{5}{2}\right)}\ln\left|\frac{u-\frac{5}{2}}{u+\frac{5}{2}}\right|\right] + C$

$= \frac{1}{2}\ln|x+2| + \frac{1}{2}\ln|x-3| - \frac{45}{2}\left[\frac{1}{5}\ln\left|\frac{x-\frac{1}{2}-\frac{5}{2}}{x-\frac{1}{2}+\frac{5}{2}}\right|\right] + C$

$= \frac{1}{2}\ln|x+2| + \frac{1}{2}\ln|x-3| - \frac{9}{2}\left[\ln|x-3| - \ln|x+2|\right] + C$

$= -4\ln|x-3| + 5\ln|x+2| + C$!

5 10pts $\int \sin(5x)\cos(3x)\,dx$

$$\left(\begin{aligned} &\sin A \cos B = \tfrac{1}{2}\left[\sin(A-B) + \sin(A+B)\right] \\ &= \tfrac{1}{2}\left[\sin(5x-3x) + \sin(5x+3x)\right] \end{aligned} \right)$$

$$= \frac{1}{2}\int \left(\sin(2x) + \sin(8x)\right)dx$$

$$= \frac{1}{4}\int \sin(2x)(2\,dx) + \frac{1}{16}\int \sin(8x)(8\,dx)$$

$$= \boxed{\frac{1}{4}\left(-\cos(2x)\right) + \frac{1}{16}\left(-\cos(8x)\right) + C}$$

$$\int \ln x\,dx \qquad \begin{aligned} u &= \ln x & dv &= dx \\ du &= \frac{1}{x}\,dx & v &= x \end{aligned}$$

$$uv - \int v\,du = x\ln x - \int x\,dx = x\ln x - \frac{x^2}{2} + C$$

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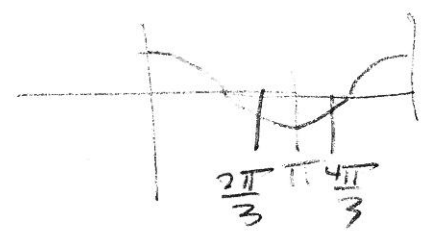
6 10 pts $\int \sin^4 x \cos^3 x \, dx$

$= \int (\sin^4 x)(1-\sin^2 x) \cos x \, dx$

$= \int \sin^4 x \cos x \, dx - \int \sin^6 x \cos x \, dx$

$\boxed{= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

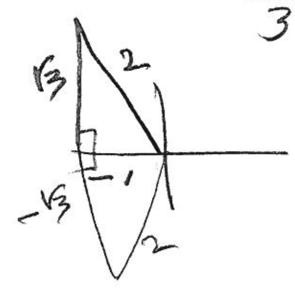
8 $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \sqrt{1-\sin x} \cdot \sqrt{\frac{1+\sin x}{1+\sin x}} \, dx$



$= \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \frac{|\cos x|}{\sqrt{1+\sin x}} \, dx = -\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \frac{\cos x}{\sqrt{1+\sin x}} \, dx$

$= -\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1+\sin x)^{-\frac{1}{2}} (\cos x \, dx) = -2(1+\sin x)^{\frac{1}{2}} \Big]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$

$= -2\left(1+\sin\frac{4\pi}{3}\right)^{\frac{1}{2}} - \left(-2\left(1+\sin\frac{2\pi}{3}\right)^{\frac{1}{2}}\right)$



$= -2\left(1-\frac{\sqrt{3}}{2}\right)^{\frac{1}{2}} - \left(-2\left(1+\frac{\sqrt{3}}{2}\right)^{\frac{1}{2}}\right)$

$= 2\sqrt{1+\frac{\sqrt{3}}{2}} - 2\sqrt{1-\frac{\sqrt{3}}{2}} = \boxed{2!}$

B3 5pts $\displaystyle\int_e^{\infty} \frac{dx}{x\ln(x)^3}$

~~$u = x$ $dv = \ln(x)^3 dx$~~
~~$du = dx$ $v =$~~

$u = \ln x \implies du = \frac{1}{x}dx$

$$= \int_e^{\infty} (\ln(x))^{-3}\left(\frac{1}{x}dx\right) = \lim_{t\to\infty} -\frac{1}{2}(\ln x)^{-2}\Big]_e^t$$

$$= \lim_{t\to\infty}\left(-\frac{1}{2}(\ln t)^{-2}\right) - \left(-\frac{1}{2}(\ln(e))^{-2}\right)$$

$$= 0 + \frac{1}{2}\cdot 1^{-2} = \boxed{\frac{1}{2}}$$

B3 5pts $\displaystyle\int_{-1}^{2\sqrt{2}-3} \frac{x+3}{\sqrt{x^2+6x+5}} = \int_{-1}^{2\sqrt{2}-3} \frac{x+3}{\sqrt{(x+3)^2-4}}dx$

$u = x+3 \longrightarrow x = -1 \rightsquigarrow u = 2,\ x = 2\sqrt{2}-3 \rightsquigarrow u = 2\sqrt{2}$

$$= \int_2^{2\sqrt{2}} \frac{u}{\sqrt{u^2-4}}du = \boxed{2 \text{ by } \#3}$$

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B4 5pts $\left(1+\frac{1}{x}\right)^{2x} = \left(\left(1+\frac{1}{x}\right)^{x}\right)^{2} \xrightarrow{x \to \infty} \boxed{e^2}$

$y = \left(1+\frac{1}{x}\right)^{2x} \longrightarrow$