MAT 202 Test 2 Spring, 2020

(1) 15 pts $\int xe^{-3x}\,dx$

$u = x \quad dv = e^{-3x}dx$

$du = dx \quad v = -\frac{1}{3}e^{-3x}$

$uv - \int v\,du = (x)(-\frac{1}{3}e^{-3x}) - \int -\frac{1}{3}e^{-3x}\,dx$

$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx =$

$u = -3x$

$du = -3dx$

$dx = -\frac{1}{3}du$

$= -\frac{1}{3}xe^{-3x} + (\frac{1}{3})(-\frac{1}{3})\int e^{-3x}(-3dx)$

$= \boxed{-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C}$

(2) 15 pts $\int x^2 \ln(\sqrt[5]{x^4})\,dx$

$u = \ln x \quad dv = x^2dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{3}x^3$

$= \frac{4}{5}\int x^2 \ln(x)\,dx$

$uv - \int v\,du = (\ln x)(\frac{1}{3}x^3) - \int (\frac{1}{3}x^3)(\frac{1}{x})\,dx$

$= \frac{1}{3}x^3 \ln(x) - \frac{1}{3}\int x^2\,dx$!

$= \boxed{\frac{1}{3}x^3 \ln(x) - \frac{1}{3} \cdot \frac{1}{3}x^3 + C}$

$= \frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + C$

3 a 10 pts $\displaystyle\int_{-\sqrt{2}}^{\sqrt{3}} x\sqrt{4-x^2}\,dx$ $\qquad u = 4-x^2$, $du = -2x\,dx$

$$= -\frac{1}{2}\int_{-\sqrt{2}}^{\sqrt{3}} (4-x^2)^{\frac{1}{2}}(-2x)\,dx = -\frac{1}{2}\int_{x=-\sqrt{2}}^{x=\sqrt{3}} u^{\frac{1}{2}}\,du$$

Pause & change back

$$= -\frac{1}{2}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{x=-\sqrt{2}}^{x=\sqrt{3}}$$

$$\left(-\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(4-x^2\right)^{3/2}\Big]_{-\sqrt{2}}^{\sqrt{3}} = -\frac{1}{3}\left[\left(4-(\sqrt{3})^2\right)^{\frac{3}{2}} - \left(4-(-\sqrt{2})^2\right)^{\frac{3}{2}}\right]$$

$$= -\frac{1}{3}\left[(4-3)^{\frac{3}{2}} - (4-2)^{\frac{3}{2}}\right] = \boxed{-\frac{1}{3}\left[1-2^{\frac{3}{2}}\right]} = \frac{1}{3}2\sqrt{2} - \frac{1}{3}$$

is fine. $-\frac{1}{3}\left[1-(2^3)^{\frac{1}{2}}\right] = -\frac{1}{3}\left[1-2\sqrt{2}\right]$

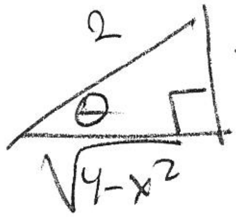
$= \dfrac{1-2\sqrt{2}}{3}$ is more stylish classical.

$\sqrt{2^3} = \sqrt{2^2\cdot 2^1} = \sqrt{2^2}\sqrt{2^1} = 2\sqrt{2}$

202 T2 3

36 10pts

$$\int_{-\sqrt{2}}^{\sqrt{3}} x\sqrt{4-x^2}\,dx$$

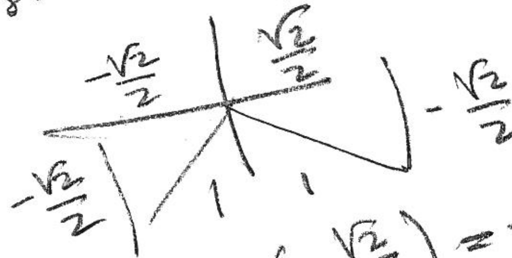


$\frac{x}{2} = \sin\theta \qquad x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

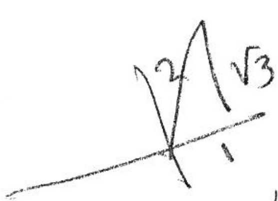
$x = 2\sin\theta = -\sqrt{2}$

$\sin\theta = -\frac{\sqrt{2}}{2}$ (or $-\frac{1}{\sqrt{2}}$)



$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45^\circ = -\frac{\pi}{4}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$ OR $\frac{\pi}{3}$



(from $2\sin\theta = \sqrt{3}$)

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} 2\sin\theta\left(\sqrt{4-(2\sin\theta)^2}\right)\left(2\cos\theta\, d\theta\right)$$

$(2\cos\theta^{*})\cos\theta\, d\theta$

$$= 4\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2(\sin\theta)(2\cos\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= 8\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \sin\theta\cos^2\theta\, d\theta$$

$$-8\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} (\cos^2\theta)(-\sin\theta\, d\theta) = -8\left[\frac{\cos^3\theta}{3}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= -\frac{8}{3}\left[\frac{(1)^3}{2^3} - \left(\frac{\sqrt{2}}{2}\right)^3\right]$$

$$= -\frac{8}{3}\left[\frac{1}{8} - \frac{2\sqrt{2}}{8}\right]$$

$$= -\frac{1}{3} + \frac{2\sqrt{2}}{3}$$

202

T2

4

4

$$\boxed{\int \frac{10x+13}{x^2-x-20}\,dx = I}$$

$$\frac{10x+13}{(x-5)(x+4)} = \frac{A}{x-5} + \frac{B}{x+4}$$

$$10x+13 = A(x+4) + B(x-5)$$

$x = -4 \Rightarrow$

$$-40+13 = -9B$$

$$\Rightarrow \frac{-27}{-9} = \boxed{B=3}$$

$x = 5 \Rightarrow$

$$10(5)+13 = 9A \rightarrow$$

$$A = \frac{\cancel{50}+13}{9} = \frac{63}{9} = \boxed{7 = A} \Rightarrow$$

$$\boxed{I = 7\ln|x-5| + 3\ln|x+4| + C}$$

202 T2 J

(5) $\int \cos(3x)\cos(11x)\,dx$

$$= \frac{1}{2}\int\left(\cos(-8x) + \cos(14x)\right)dx$$

$$\boxed{= \frac{1}{2}\left[\frac{\sin(8x)}{8} + \frac{\cos(14x)}{14}\right] + C}$$

$u = 8x$
$du = 8\,dx$
$dx = \frac{du}{8}$

$u = 14x$
$du = 14\,dx$
$dx = \frac{du}{14}$

(6) Start fresh page. It's YUGE.

(6) (10 pts) $\int \sin^4 x \, dx = I$

$\sin^4 x = (\sin^2 x)^2 = \left(\frac{1-\cos(2x)}{2}\right)^2 = \frac{(\cos(2x)-1)^2}{4}$

$= \frac{1}{4}\left(\cos^2(2x) - 2\cos(2x) + 1\right)$

$= \frac{1}{4}\left(\frac{\cos(4x)+1}{2} - 2\cos(2x) + 1\right) \longrightarrow$

$I = \int \left(\frac{1}{8}\cos(4x) + \frac{1}{8} - \frac{1}{2}\cos(2x) + \frac{1}{4}\right) dx$

$\boxed{= \frac{1}{8}\cdot\frac{1}{4}\sin(4x) + \frac{3}{8}x - \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C}$

$= \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x) + \frac{3}{8}x + C$

202 T2 M (7)

B1 5 pts

$$\int_1^{\sqrt{3}} \frac{\sqrt{4-x^2}}{x^2}\,dx \qquad a=2,\ u=x$$

$\boxed{\#33}$

$$\int_1^{\sqrt{3}} \frac{\sqrt{4-x^2}}{x^2}\,dx = \left[-\frac{1}{x}\sqrt{2^2-x^2} - \sin^{-1}\left(\frac{x}{2}\right)\right]_1^{\sqrt{3}}$$

$$= -\frac{1}{\sqrt{3}}\sqrt{4-\sqrt{3}^2} - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \left[-\frac{1}{1}\sqrt{4-1^2} - \sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= -\frac{1}{\sqrt{3}}\sqrt{1} - \frac{\pi}{3} + \sqrt{3} + \frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{3} - \frac{\pi}{6} + \frac{3\sqrt{3}}{3} = \frac{2\sqrt{3}}{3} - \frac{\pi}{6}$$

$$\int_1^{\sqrt{3}} \frac{\sqrt{4-x^2}}{x^2}\,dx$$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

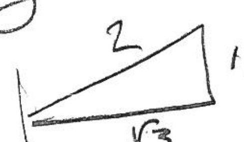
$2\sin\theta = \sqrt{3}$ $\quad 2\sin\theta = 1$

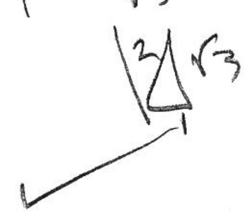
$\sin\theta = \frac{\sqrt{3}}{2}$ $\quad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$ $\quad \theta = \frac{\pi}{6}$

$$= \int_{x=1}^{x=\sqrt{3}} \frac{\sqrt{4-4\sin^2\theta}}{4\sin^2\theta}\,2\cos\theta\,d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\cos\theta \cdot 2\cos\theta\,d\theta}{4\sin^2\theta} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1-\sin^2\theta}{\sin^2\theta}\,d\theta$$





$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\csc^2\theta - 1)\,d\theta = -\cot\theta - \theta\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= -\frac{1}{\sqrt{3}} - \frac{\pi}{3} - \left[-\sqrt{3} - \frac{\pi}{6}\right] = \frac{2\sqrt{3}}{3} - \frac{\pi}{6}$$

B2 5pts $\displaystyle\int \frac{dx}{x(\ln x)^4}$ $u = \ln x$, $du = \frac{1}{x}dx$

$$= \int (\ln(x))^{-4}\left(\frac{1}{x}dx\right) = \boxed{\frac{1}{5}(\ln(x))^5 + C}$$

B3 5pts $y = (1+x)^{\frac{3}{x}} \longrightarrow$

$$\ln y = \frac{3}{x}\ln(1+x) \xrightarrow{x \to 0} \infty \cdot 0$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{\left(\frac{x}{3}\right)} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{x+1}}{\frac{1}{3}} = \lim_{x\to 0} \frac{3}{x+1} = 3$$

$$\Longrightarrow \lim_{x\to 0} y = \boxed{e^3}$$

B4 5pts $\int e^x \sin x\, dx = I$

$u = e^x$ $du = e^x dx$ $dv = \sin x\, dx$ $v = -\cos x$

$u = e^x$ $du = e^x dx$ $dv = \cos x\, dx$ $v = \sin x$

$u = e^x$ $du = e^x dx$ $dv = \cos x\, dx$

$$= e^x(-\cos x) + \int e^x \cos x\, dx$$

$$= -e^x \cos x + \left[e^x(\sin x) - \int e^x \sin x\, dx\right] = -e^x\cos x - e^x \sin x - I$$

$$= I \Longrightarrow 2I = e^x \sin x - e^x \cos x + \hat{C}$$

Different constant

$$\Longrightarrow I = \boxed{\frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C}$$