MAT 202
100 Points

Test 1, Spring, 2020
Covers Chapter 6

(5 pts) Name KEY

Do all your work and put all your answers WITH your work, CIRCLED, on the white paper provided. All I want on this sheet is your NAME! Spend no more than 2 minutes on any single problem on your first pass through the test. If you don't finish a problem in 2 or 3 minutes, start a fresh sheet of paper for the next problem, and so on.

Formatting should be the same as homework, only you don't need to re-state the question, because the question's attached to your test!

1. The function $f(x)=x^2-7x-15$ is 1-to-1 on the restricted domain $D=\left[\frac{7}{2},\infty\right)$.
   a. (10 pts) Find the inverse function $f^{-1}(x)$. State its domain and range.
   b. (5 pts) Find $\left(f^{-1}\right)'(5)$, directly, by differentiating your answer for part a.
   c. (5 pts) Find $\left(f^{-1}\right)'(5)$ by applying a theorem regarding derivatives of inverse functions.

2. (5 pts each) Find the derivative with respect to x. All "-1" powers refer to function inverses, not reciprocals.
   a. $y=3\cdot 2^{\sin(x)}$
   b. $y=\ln\left(\dfrac{\sqrt[5]{x^2-3x}}{\sin^3(x)}\right)$
   c. $y=\log_5\left(\tan\left(x^2\right)\right)$
   d. $y=\left[7x^3-5x\right]^{\cos(x)}$
   e. $y=\cos(x)\cdot\sin^{-1}\left(5x^3-7x\right)$ or $\cos(x)\cdot\arcsin\left(5x^3-7x\right)$
   f. $y=\sin(x)\cdot\cosh^{-1}\left(5x^3-7x\right)$

3. (5 pts each) Evaluate the integrals
   a. $\displaystyle\int \sec^2(x)\cdot e^{\tan(x)}\,dx$
   b. $\displaystyle\int \frac{dx}{5x\sqrt{x^2-36}}$

4. (5 pts each) Simplify the following.
   a. $\sec\left(\tan^{-1}\left(\sqrt{9x^2-100}\right)\right)$
   b. $\sin^{-1}\left(\sin\left(\dfrac{5\pi}{4}\right)\right)$. I *think* you're OK on the domains, after class talk.

5. (10 pts) The doubling time of an investment is 10 years. Assuming interest compounds continuously, what is the rate of interest?

6. (5 pts each) Evaluate the following limits:
   a. $\lim_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x}$
   b. $\lim_{x\to 0}\left(\dfrac{e^{2x}-1}{\sin(x)}\right)$
   c. $\lim_{x\to\frac{\pi}{2}^-}\left(\sec(x)-\tan(x)\right)$

Bonus:

1. Find the volume of the solid of revolution obtained by revolving the function $y = \sqrt{x}$ about the y-axis in 2 ways:
   a. (10 pts) Shell Method
   b. (10 pts) Disk Method

1 $f(x) = x^2 - 7x - 15$ on $\left[\frac{7}{2}, \infty\right)$

$= x^2 - 7x + \left(\frac{7}{2}\right)^2 - \frac{49}{4} - \frac{60}{4}$

a 10 pts

$= \left(x - \frac{7}{2}\right)^2 - \frac{109}{4}$ $(h, k) = \left(\frac{7}{2}, -\frac{109}{4}\right)$

SET $= y \longrightarrow$

$\left(x - \frac{7}{2}\right)^2 = y + \frac{109}{4} \longrightarrow$

$x - \frac{7}{2} = \pm\sqrt{y + \frac{109}{4}} \longrightarrow$

$x = \frac{7}{2} \pm \sqrt{y + \frac{109}{4}} \longrightarrow \boxed{f^{-1}(x) = \frac{7}{2} + \sqrt{x + \frac{109}{4}}}$

$$\boxed{\begin{aligned} \mathcal{D}(f^{-1}) &= \mathcal{R}(f) = \left[-\frac{109}{4}, \infty\right) \\ \mathcal{R}(f^{-1}) &= \mathcal{D}(f) = \left[\frac{7}{2}, \infty\right) \end{aligned}}$$

b 5 pts $(f^{-1})'(x) = \frac{1}{2}\left(x + \frac{109}{4}\right)^{-\frac{1}{2}} \longrightarrow$

$(f^{-1})'(5) = \frac{1}{2}\left(5 + \frac{109}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{129}{4}\right)^{-\frac{1}{2}}$

$= \left(\frac{1}{2}\right)\frac{2}{\sqrt{129}} = \boxed{\frac{1}{\sqrt{129}} = (f^{-1})'(5)}$

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1c 5pts $(f^{-1})'(5) = \dfrac{1}{f'(f^{-1}(5))} = \dfrac{1}{2f^{-1}(5)-7} \rightarrow$

b/c $f' = 2x-7$. Need

$f^{-1}(5)$: $f(x) = 5$

$\rightarrow x^2 7x - 15$

$= (x-\frac{7}{2})^2 - \frac{109}{4} = 5 = \frac{20}{4} \longrightarrow$

$(x-\frac{7}{2})^2 = \frac{129}{4} \longrightarrow$

$x - \frac{7}{2} = \pm \frac{\sqrt{129}}{2} \longrightarrow$

$x = \frac{7}{2} \pm \frac{\sqrt{129}}{2}$ Take the positive

$(f^{-1})'(5) = \dfrac{1}{2(\frac{7+\sqrt{129}}{2})-7} =$ $\boxed{\dfrac{1}{\sqrt{129}} ! = (f^{-1})'(5)}$

2 a 5 pts ∀ parts

$y = 3 \cdot 2^{\sin x} \to \boxed{y' = (3 \ln 2 \cdot 2^{\sin x}) \cos x}$

b $y = \ln\left(\dfrac{\sqrt[5]{x^2 - 3x}}{\sin^3 x}\right) = \frac{1}{5}\ln(x^2 - 3x) - 3\ln(\sin x)$

$\to \boxed{y' = \frac{1}{5}\left(\dfrac{2x-3}{x^2 - 3x}\right) - 3\,\dfrac{\cos x}{\sin x}}$

c $y = \log_5(\tan(x^2)) \to$

$\boxed{y' = \dfrac{1}{\ln 5}\left(\dfrac{\sec^2(x^2)(2x)}{\tan(x^2)}\right)}$

d $y = (7x^3 - 5x)^{\cos x} \to$

$\ln y = (\cos x)\ln(7x^3 - 5x) \to$

$\dfrac{y'}{y} = -\sin x \ln(7x^3 - 5x) + (\cos x)\left(\dfrac{21x^2 - 5}{7x^3 - 5x}\right) \to$

$\boxed{y' = \left((-\sin x)\ln(7x^3 - 5x) + (\cos x)\left(\dfrac{21x^2 - 5}{7x^3 - 5x}\right)\right)(7x^3 - 5x)^{\cos x}}$

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(4)

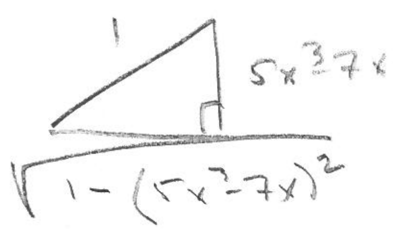
(e) $y = (\cos x)(\sin^{-1}(5x^3 - 7x)) \longrightarrow$

$$\boxed{y' = (-\sin x)(\sin^{-1}(5x^3 - 7x)) + (\cos x)\left(\frac{15x^2 - 7}{\sqrt{1 - (5x^3 - 7x)^2}}\right)}$$

$$\frac{1}{\cos(\sin^{-1}(5x^3 - 7x))}$$

$f(x) = \sin x \quad f'(x) = \cos x$, f

$$f'(f^{-1}(x)) = \cos(\sin^{-1}(5x^3 - 7x))(15x^2 - 7)$$



$$= \sqrt{1 - (5x^3 - 7x)^2}$$

$$\boxed{f'(g(x)) = \frac{df}{dg} \cdot \frac{dg}{dx}}$$

$$\frac{df(g(x))}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

(3) (5 pts each) (a) $\int \sec^2 x \, e^{\tan x} dx = \boxed{e^{\tan x} + C}$

$u = \tan x, \; du = \sec^2 x \, dx$

(b) $\overline{}$
$5x$

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TEST 1

5

3b

$$\int \frac{dx}{5x\sqrt{x^2-36}} = \frac{1}{5}\int \frac{dx}{x\cdot 6\sqrt{\frac{x^2}{36}-1}}$$

$$= \frac{1}{30}\int \frac{dx}{x\sqrt{\left(\frac{x}{6}\right)^2-1}} \qquad u=\frac{x}{6} \Rightarrow x=6u \text{ \&} \quad dx = 6du$$

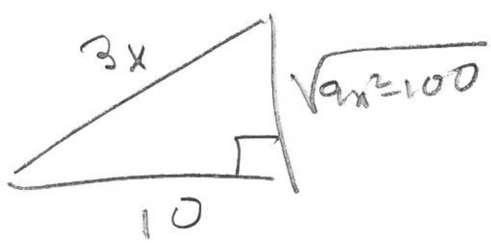
$$= \frac{1}{30}\int \frac{6du}{6u\sqrt{u^2-1}} = \frac{1}{30}\sec^{-1}(u)+C$$

$$\boxed{= \frac{1}{30}\sec^{-1}\left(\frac{x}{6}\right)+C}$$

4 5pts ea

2

$$\sec\left(\tan^{-1}\left(\frac{\sqrt{9x^2-100}}{10}\right)\right) = \sec\theta$$



$$\boxed{= \frac{3x}{10}}$$

6

$$\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-\frac{\pi}{4}}$$

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5 10 pts Doubling time is 10 yrs.

What's the interest rate?

Let $A(t)$ = Accumulated amt in \$

as a function of...

t = time in years

Then $A(t) = A_0 e^{kt}$, where A_0 = initial amt and $k = r$ = relative rate of growth. Then

$$A_0 e^{10k} = 2A_0 \longrightarrow$$

$$e^{10k} = 2$$

$$10k = \ln 2$$

$$k = \frac{\ln 2}{10} \approx .0693147 18$$

or $\boxed{6.931471 8\% = r}$

6a 5pts ea

$$y = \left(1+\frac{3}{x}\right)^{5x} \longrightarrow$$

$$\ln y = 5x \ln\left(1+\frac{3}{x}\right) \longrightarrow$$

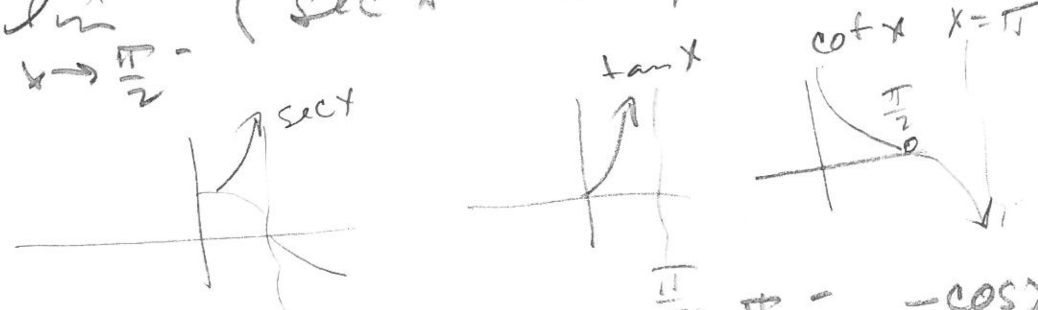
$$\frac{y'}{y} = 5\ln\left(1+\frac{3}{x}\right) + (5x)\left(\frac{-\frac{3}{x^2}}{1+\frac{3}{x}}\right) \longrightarrow$$

$$\boxed{y' = \left(5\ln\left(1+\frac{3}{x}\right) - \left(\frac{15}{x}\right)\left(\frac{1}{1+\frac{3}{x}}\right)\right)\left(1+\frac{3}{x}\right)^{5x}}$$

b $$\lim_{x\to 0}\left(\frac{e^{2x}-1}{\sin x}\right) \overset{L'H}{=} \lim_{x\to 0} \frac{2e^{2x}}{\cos x} = \frac{2}{1} = \boxed{2}$$

$\frac{0}{0}$

c $$\lim_{x\to \frac{\pi}{2}^-} (\sec x - \tan x) = \infty - \infty, \text{ so}$$



$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} = \frac{1-\sin x}{\cos x} \xrightarrow[L'H]{x \to \frac{\pi}{2}^-} \frac{-\cos x}{-\sin x}$$

$$= \cot x \xrightarrow{x \to \frac{\pi}{2}^-} \boxed{0}$$

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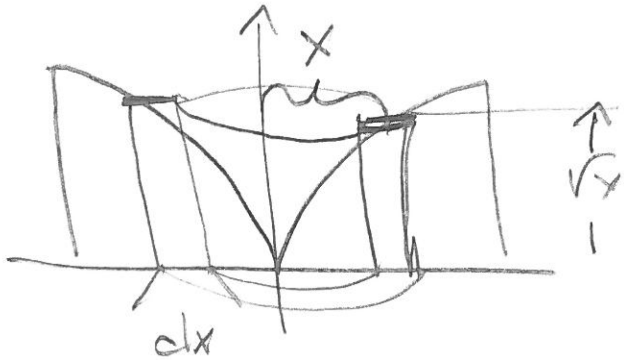
Bonus a 10 pts

$$2\pi \int_0^1 x\sqrt{x}\, dx$$

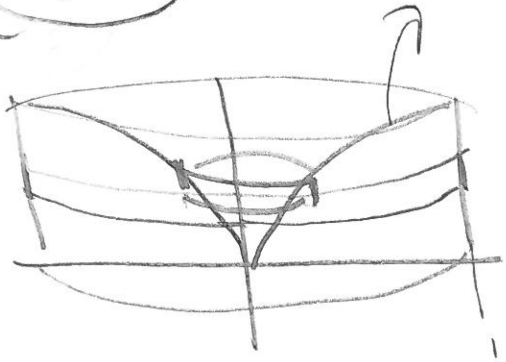


$$= 2\pi \int_0^1 x^{\frac{3}{2}} dx$$

$$= (2\pi)\left(\frac{2}{5}\right) x^{\frac{5}{2}} \Big]_0^1$$

$$\boxed{= \frac{4\pi}{5}}$$

b 10 pts



$$y = \sqrt{x} \Rightarrow x = y^2$$

$$V = \pi \int_0^1 \left(1^2 - (y^2)^2\right) dy$$

$$= \pi \int_0^1 (1 - y^4) dy$$

$$= \pi \left[y - \frac{1}{5} y^5 \right]_0^1 = \pi\left(1 - \frac{1}{5}\right) =$$

$$= \pi\left(\frac{4}{5}\right) = \boxed{\frac{4\pi}{5}}$$