Today: Binomial Series

Maybe some term-by-term products/quotients, like $e^x \sin(x)$, which was done in the textbook.

Recall Test 2 Take-Home $\int_{-2}^{6} e^{-x^2}\,dx$ $\quad \tan x = \frac{\sin x}{\cos x}$

Example in Book

$$\sqrt[3]{1-x} = (1-x)^{\frac{1}{3}} = (1+(-x))^{\frac{1}{3}}$$

Look for products that allow you to take notes with the stylus.

$$(1+(-x))^{\frac{1}{3}}$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

$$(1+(-x))^k = \sum_{n=0}^{\infty} \binom{k}{n} (-x)^n$$

$$\boxed{0! = 1}$$

$$\binom{\frac{1}{3}}{0} = 1 \qquad \frac{1}{3}$$

$n=0$

$$\frac{k(k-1)(k-2)\cdots(k-n+1)}{n!} =$$

$$\binom{\frac{1}{3}}{1} = \frac{\left(\frac{1}{3}\right)}{1!}$$

$$\binom{\frac{1}{3}}{2} = \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2!} = \frac{-\frac{2}{9}}{2!}$$

$$\binom{\frac{1}{3}}{3} = \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!} = \frac{\frac{2\cdot 5}{3^3}}{3!}$$

3n switch $\qquad$ 3n−4

3n−1

$$\binom{\frac{1}{3}}{4} = \frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)}{4!} = \frac{-\frac{2\cdot 5\cdot 8}{3^4}}{4!} \qquad \boxed{(-1)^{n+1}\frac{2\cdot 5\cdot 8\cdots(3(n-1)-1)}{3^n\, n!}}$$

The $n^{\text{th}}$ coefficient.

$$(1-x)^{\frac{1}{3}} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(2)(5)\cdots(3n-4)}{3^n\, n!}(-x)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2\cdot 5\cdots(3n-4)}{3^n\, n!} x^n$$

Term-by-term Products & Quotients.

$e^x \sin x$

$$\frac{1}{1-x}e^x = \left(\sum_{n=0}^{\infty} x^n\right)\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)$$

$$= \left(1 + x + x^2 + x^3 + x^4 + \dots\right)\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots\right)$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

$$x + x^2 + \frac{x^3}{2} + \frac{x^4}{6} + \frac{x^5}{24} + \dots$$

$$x^2 + x^3 + \frac{x^4}{2} + \frac{x^5}{6} + \frac{x^6}{24}$$

$$x^3 + x^4 + \frac{x^5}{2} + \frac{x^6}{6} + \frac{x^7}{24}$$

$\frac{3}{3}\frac{5}{2} + \frac{1}{6} = \frac{15+1}{6} = \frac{16}{6} = \frac{8}{3}$

$$1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \dots$$

$$\frac{3x - \sin(3x)}{3x^3} =$$

$$\sin(3x) = \sum_{n=0}^{\infty} (-1)^n \frac{(3x)^{2n+1}}{(2n+1)!} = \frac{(-1)^0 (3x)^1}{1} + \frac{(-1)^1 (3x)^3}{3!} = 3x - \frac{27x^3}{6}$$

$$= 3x - \frac{9}{2}x^3$$

$$\text{So} \quad 3x - \sin(3x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(3x)^{2n+1}}{(2n+1)!}$$

$$n \quad = \sum_{n=0}^{\infty} (-1)^n \frac{(3x)^{2(n+1)+1}}{(2(n+1)+1)!} = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{(3x)^{2n+3}}{(2n+3)!}$$

$$\frac{3x - \sin(3x)}{3x^3} = \frac{\sum_{n=0}^{\infty} (-1)^{n+1} \frac{3^{2n+3} x^{2n+3}}{(2n+3)!}}{3x^3} = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{3^{2n+2} x^{2n}}{(2n+3)!}$$

$n$

$$3^{2n+2} = 3^{2n} \cdot 3^2 = 9 \cdot 3^{2n}$$

$$= 9 \sum (-1)^n \frac{3^{2n} x^{2n}}{(2n+3)!}$$

$$= 9 \sum_{n=0}^{\infty} (-1)^n \frac{(3x)^{2n}}{(2n+3)!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = 1 - \frac{x^2}{1!} + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{3!}$$

$$= 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} + \cdots$$

$$\int_{-2}^{6} e^{-x^2}\,dx = \left[1 - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots\right]_{-2}^{6}$$

Apr 24-10:13 AM