Chapter 11 Questions?

2.5 things remaining

1. Maclaurin / Taylor Series
2. Binomial Series Toughest for me.
2.5 Taylor's Inequality

$$f(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n + \cdots = \sum_{n=0}^{\infty} c_n x^n$$

$f(0) = c_0$

$f'(x) = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + \cdots$

$f'(0) = c_1$

$f''(x) = 2c_2 + 6c_3 x + 12 c_4 x^2 + 20 c_5 x^3 + \cdots$

$f''(0) = 2c_2$

$f'''(0) = 3\cdot 2\, c_3$

$f^{(iv)}(0) = 4\cdot 3\cdot 2\, c_4$

$f^{(v)}(0) = 5\cdot 4\cdot 3\cdot 2\, c_5$

$c_0 = f(0)$

$c_1 = f'(0) = \frac{f'(0)}{1!}$

$c_2 = \frac{f''(0)}{2!}$

$c_3 = \frac{f'''(0)}{3!}$

If $f(x)$ HAS a power series representation, then $f(x)$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Maclaurin Series.

Taylor Series Generalizes THIS:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$ Expanded around/about/at $x = a$.

$$c_n = \frac{f^{(n)}(a)}{n!}$$

Maclaurin's is just Taylor's w/ $a = 0$.

Using Taylor's Ideas to Build a power series.

$f(x) = \sin(x)$

$f(0) = 0$ $\qquad n=0$

$f'(x) = \cos x \quad f'(0) = \cos(0) = 1 \qquad n=1 \quad +1 \qquad 2n+1$

$f''(x) = -\sin x \quad \sin(0) = 0$

$f'''(x) = -\cos x \quad f''(0) = -1 \qquad n=3 \quad -1$

f ⋮ $\qquad n=5 \quad +1$

$\qquad n=7 \quad -1$

$f^{(2n+1)}(x) = \qquad n=9 \quad +1$

$$\sum \frac{f^{(n)}(0)}{n!} x^n = 1x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$S = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Error Estimates:

Alternating is nice, when you can get it.

$|E_n| = |S - S_n|$ = error of the n^{th} partial sum: $|n\text{-tail}|$

$$|E_n| = \left| \sum_{k=0}^{\infty} - \sum_{k=0}^{n} \right|$$

$$= \left| \sum_{k=n+1}^{\infty} a_k (x-a)^k \right| \leq |a_{n+1}| \, |x-a|^{n+1}$$

In general, $|E_n| \leq \dfrac{M\,|x-a|^{n+1}}{(n+1)!}$, where

$M \geq |f^{(n+1)}(x)|$

where $x \in$ Interval in question.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$$g(x) = \frac{1}{(1-x)^3}$$

$$f'(x) = \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 + \dots$$

$$f''(x) = -2(1-x)^{-3}(-1) = \frac{2}{(1-x)^3} = 2g(x)$$

$$f''(x) = 2 + 6x + 12x^2 + 20x^3 + 30x^4 + 42x^5 + 56x^6 + \dots$$

$$\cancel{g(x) = \frac{1}{2} f''(x) = \frac{1}{2}\left[2 + 6x + 12x^2 + 20x^3 + \dots\right] = 1 + 3x + 6x^2 + 10x^3}$$

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$$

$$\cancel{= \frac{1}{2}\sum_{n=0}^{\infty} \qquad = \sum_{n=0}^{\infty}(n+2)x^n}$$

$$f''(x) = 2 + 6x + 12x^2 + 20x^3 + 30x^4 + 42x^5 + 56x^6 + \dots$$

$$= 2g(x) \Rightarrow g(x) = \frac{1}{2}\left[\ \uparrow\ \right] = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \dots$$

$$\begin{array}{cccc} 3\cdot 1 & 3\cdot 2 & 5\cdot 2 & 5\cdot 3 \end{array}$$

$$g(x) = \frac{1}{2} f''(x) = \frac{1}{2}\left[2\cdot 1 + 3\cdot 2x + 4\cdot 3x^2 + 5\cdot 4x^3 + \dots\right]$$

$$\frac{1}{2}\left[\sum_{n=0}^{\infty}(n+1)(n+2)x^n\right] = \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$$

where does this converge?
Gimme the radius of convergence.

$$\left| \frac{\frac{(n+2)(n+3)}{2} x^{n+1}}{\frac{(n+1)(n+2)}{2} x^n} \right| = \frac{n+3}{n+1}|x| \quad \overset{\text{want}}{<} 1 \equiv R$$

$$\frac{1}{1+x^7} = \frac{1}{1-(-x^7)} = \sum_{n=0}^{\infty} (-x^7)^n = \sum_{n=0}^{\infty} (-1)^n x^{7n}$$

Next time: Binomial Expansion.

$$(x+1)^5 = \binom{5}{0} 1 + \binom{5}{1} x + \binom{5}{2} x^2 + \binom{5}{3} x^3 + \binom{5}{4} x^4 + \binom{5}{5} x^5$$

$$= \binom{5}{0} x^5 + \binom{5}{1} x^4 + \binom{5}{2} x^3 + \binom{5}{3} x^2 + \binom{5}{4} x + \binom{5}{5} 1$$

$$\binom{5}{2} = \binom{5}{5-3} = \binom{5}{3}$$

Binomial Coefficient:

$C(k,n)$ = # of n-member subsets of a set of k things.

"k choose n" when k & n are positive integers & $k \geq n$

$$\binom{k}{n} = \frac{k!}{(k-n)!\,n!}$$

$$\binom{5}{3} = \frac{5!}{2!\,3!} = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{(2\cdot 1)\cancel{(3\cdot 2\cdot 1)}} = 10$$

$$\binom{5}{2} = \frac{5!}{3!\,2!} = 10$$

Pascal's Triangle's Symmetry.

COMBINATIONS : CHOOSE!

* PERMUTATIONS : CHOOSE! ARRANGE! *

$$P(k,n) = \frac{k!}{(k-n)!} \qquad C(k,n) = \frac{k!}{(k-n)!\,n!}$$

The n! is the # of rearrangements of the things after you choose them.

$$C(n,k) = \frac{k!}{(k-n)!\,n!} = \frac{k\cdot(k-1)(k-2)\cdots\cancel{(k-n)(k-n-1)(k-n-2)\cdots}}{\cancel{(k-n)(k-n-1)\cdots}\,(n\cdot(n-1)(n-2)\cdots)}$$

$$= \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!}$$

Apr 22-9:52 AM

MAKE FINAL AVAILABLE NEXT WEEK &
LEAVE IT OPEN 'TIL THE END.

LAB OPEN FOR AT LEAST 20 minutes.
So enjoy!

970-290-0550

$$\frac{1}{3+x} = \frac{1}{3(1+\frac{x}{3})} = \frac{1}{3}\left[\frac{1}{1+\frac{x}{3}}\right]$$

$$= \frac{1}{3}\left[\frac{1}{1-(-\frac{x}{3})}\right] =$$

$$\sum (-1)^n x^n = \frac{1}{1+x}$$

$$\frac{1}{3}\sum (-1)^n \left(\frac{x}{3}\right)^n = \frac{1}{3}\left[\frac{1}{1+\frac{x}{3}}\right]$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$\frac{1}{3}\left[\frac{1}{1-(-\frac{x}{3})}\right] = \frac{1}{3}\left[1 + (-\tfrac{x}{3}) + (-\tfrac{x}{3})^2 + (-\tfrac{x}{3})^3 + \dots\right]$$

$$= \frac{1}{3}\left[1 - \frac{x}{3} + \frac{x^2}{3^2} - \frac{x^3}{3^3} + \dots\right]$$

$$= \frac{1}{3} - \frac{x}{3^2} + \frac{x^2}{3^3} - \frac{x^3}{3^4} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} x^n$$

$$\sum a_n x^n \qquad \sum a_n (x-a)^n$$

$$\left| \frac{x^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{x^n} \right|$$

$$= \left| \tfrac{1}{3} x \right| < 1$$

$$|x| < 3$$

$$-3 < x < 3$$

WebAssign appears to disagree! $R=3$ &

$I = (-3, 3]$, according to my work!