Let me know where you are in the homework.

Use chat. Public/private is fine.

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https://harryzaims.com/202/videos/chapter-11/11-08-power-series/11-8-notes.pdf

https://harryzaims.com/202/videos/chapter-11/11-10/11-10-notes.pdf *

* Pg 11, Binomial Coefficient:

$\binom{k}{n} = \frac{k!}{n!(k-n)!}$ not $\frac{k!}{n!(n-k)!}$

I learned $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Recall Geometric Sums.

$$a+ar+ar^2+\dots+ar^{n-1} = \sum_{k=1}^{n} ar^{k-1}$$

We showed this is

$$a\left(\frac{1-r^n}{1-r}\right) \xrightarrow{n\to\infty} a\left(\frac{1}{1-r}\right) \text{ if } -1<r<1 \quad (|r|<1)$$

$$\frac{1}{1-x} = 1+x+x^2+x^3+\dots+x^n+\dots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$(-x)^n = ((-1)x)^n = (-1)^n x^n$$

Spin-offs like crazy $\quad \frac{1}{1+x^3} = \frac{1}{1-(-x^3)} = \sum_{n=0}^{\infty} (-x^3)^n$

$$= \sum_{n=0}^{\infty} (-1)^n x^{3n}$$

ALSO, Term-by-term $\int dx$ & $\frac{d}{dx}$.

$f(x) = \ln(1-x) = ?$

Notice $\frac{d}{dx}\left[\ln(1-x)\right] = \frac{-1}{1-x} = -\left(\frac{1}{1-x}\right)$

$$= -\sum_{n=0}^{\infty} x^n = f'(x) \text{, so } \ln(1-x) = -\int \frac{dx}{1-x}$$

$$= -\sum_{n=0}^{\infty} \frac{d}{dx}[x^n] = -\sum_{n=1}^{\infty} nx^{n-1} = \sum_{n=0}^{\infty} (n+1)x^n$$

$1+x+x^2+x^3+\dots$

$1+2x+3x^2+4x^3+\dots$

$\frac{d}{dx}\left[-\sum x^n\right] = -\sum \frac{d}{dx}[x^n]$, because $\frac{d}{dx}$ is a linear operator

$f(x) = \frac{1}{1+x} = (1+x)^{-1}$

$f'(x) = -(1+x)^{-2}$

$f''(x) = 2(1+x)^{-3}$

$\boxed{f'''(x) = -6(1+x)^{-4}}$

etc.

Suggests a way of finding a powerseries for $\frac{1}{(1+x)^4}$

Integrating, term by term:

Recall $\arctan(x) + C = \int \frac{1}{1+x^2}\,dx$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-1)^n(-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n} = f'(x)!$$

$$\Rightarrow \arctan(x) = \sum_{n=0}^{\infty} (-1)^n \int x^{2n}\,dx = \left(\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}\right) + C = \arctan(x)$$

$\arctan(0) = 0$, so $C=0$.

All the $\int \frac{dx}{a^2 \pm x^2}$

$\frac{1}{a+bu}$? Nah. $\frac{1}{a(1+\frac{b}{a}u)} = \frac{1}{a(1+v)}$, where $v=\frac{b}{a}u$

$\sum_{n=2}^{\infty} \frac{x^{n+2}}{\sqrt{n}}$ Convergence:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+3}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^{n+2}}\right| = \frac{\sqrt{n}}{\sqrt{n+1}}|x| \left(\begin{matrix}\text{want} \\ < 1 \text{ in} \\ \text{the limit}\end{matrix}\right)$$

$\xrightarrow{n \to \infty} |x| < 1 \Rightarrow \boxed{R=1}$

$x=1$: $\sum \frac{(1)^{n+2}}{\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$ $\not\longrightarrow$

$x=-1$: $\frac{(-1)^{n+2}}{\sqrt{n}} = \frac{(-1)^n(-1)^2}{\sqrt{n}} = \frac{(-1)^n}{\sqrt{n}}$ & $\sum \frac{(-1)^n}{\sqrt{n}}$ is

alternating series whose terms converge to zero

$\Rightarrow I = [-1, 1)$

Next time: Binomial expansion!

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & \binom{1}{0} & & \binom{1}{1} & & & \\
 & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & \\
 & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & \\
\binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4}
\end{array}
$$

$$(x+1)^4 = (1+x)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$$
$$= \binom{4}{0}x^4 + \binom{4}{1}x^3 + \binom{4}{2}x^2 + \binom{4}{3}x^1 + \binom{4}{4}$$
$$= \sum_{k=0}^{4}\binom{4}{k}x^k$$

Binomial Coefficient

$$\sqrt{1+x} = ?$$
$$(1+x)^{\frac{1}{2}} = \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}x^k$$