

$$f := x \rightarrow \exp(-x^2)$$

$$f := x \mapsto e^{-x^2} \tag{1}$$

$$\int_{-2}^6 \exp(-x^2) dx$$

$$\frac{\operatorname{erf}(2) \sqrt{\pi}}{2} + \frac{\operatorname{erf}(6) \sqrt{\pi}}{2} \tag{2}$$

evalf(%)

$$1.768308316 \tag{3}$$

$$\text{right_sum} := \frac{8}{n} \cdot \sum_{k=1}^n f\left(-2 + \frac{k \cdot 4}{5}\right)$$

$$\text{right_sum} := \frac{4 \left(\sum_{k=1}^n e^{-\left(-2 + \frac{4k}{5}\right)^2} \right)}{5} \tag{4}$$

$$\text{left_sum} := \frac{8}{n} \cdot \sum_{k=0}^{n-1} f\left(-2 + \frac{k \cdot 4}{5}\right)$$

$$\text{left_sum} := \frac{8 \left(\sum_{k=0}^{n-1} e^{-\left(-2 + \frac{4k}{5}\right)^2} \right)}{n} \tag{5}$$

left_sum - *right_sum*

$$\frac{8 \left(\sum_{k=0}^{n-1} e^{-\left(-2 + \frac{4k}{5}\right)^2} \right)}{n} - \frac{4 \left(\sum_{k=1}^n e^{-\left(-2 + \frac{4k}{5}\right)^2} \right)}{5} \tag{6}$$

simplify(%)

$$\left(\sum_{k=0}^{n-1} e^{-\frac{4(-5+2k)^2}{25}} \right) - \frac{4 \left(\sum_{k=1}^n e^{-\frac{4(-5+2k)^2}{25}} \right)}{5} \tag{7}$$

$$\Delta x := \frac{4}{5}$$

$$\Delta x := \frac{4}{5} \tag{8}$$

$$a := -2$$

$$a := -2 \tag{9}$$

$$b := 6$$

$$b := 6 \tag{10}$$

$$\frac{\Delta x}{3} \cdot (f(a) + 4 \cdot f(a + \Delta x) + 2 \cdot f(a + 2 \cdot \Delta x) + 4 \cdot f(a + 3 \cdot \Delta x) + 2 \cdot f(a + 4 \cdot \Delta x) + 4 \cdot f(a + 5 \cdot \Delta x) + 2 \cdot f(a + 6 \cdot \Delta x) + 4 \cdot f(a + 7 \cdot \Delta x) + 2 \cdot f(a + 8 \cdot \Delta x) + 4 \cdot f(a + 9 \cdot \Delta x) + f(a + 10 \cdot \Delta x))$$

$$\frac{4 e^{-4}}{3} + \frac{8 e^{-\frac{36}{25}}}{5} + \frac{8 e^{-\frac{4}{25}}}{5} + \frac{8 e^{-\frac{196}{25}}}{15} + \frac{16 e^{-\frac{324}{25}}}{15} + \frac{8 e^{-\frac{484}{25}}}{15} + \frac{16 e^{-\frac{676}{25}}}{15} + \frac{4 e^{-36}}{15} \quad (11)$$

evalf(%)

$$1.767147796 \quad (12)$$

$a + 10 \cdot \Delta x$

$$6 \quad (13)$$

`ApproximateInt(exp(-x^2), -2..6, 'partition' = 10, 'method' = simpson, 'partitiontype' = normal, 'output' = 'value', 'boxoptions' = ['filled' = ['transparency' = .5]]);`

$$\frac{4 e^{-4}}{3} + \frac{8 e^{-\frac{36}{25}}}{5} + \frac{8 e^{-\frac{4}{25}}}{5} + \frac{8 e^{-\frac{196}{25}}}{15} + \frac{16 e^{-\frac{324}{25}}}{15} + \frac{8 e^{-\frac{484}{25}}}{15} + \frac{16 e^{-\frac{676}{25}}}{15} + \frac{4 e^{-36}}{15} \quad (14)$$

evalf(%)

$$1.767147796 \quad (15)$$

$$\int_{-2}^6 \exp(-x^2) dx$$

$$\frac{\operatorname{erf}(2) \sqrt{\pi}}{2} + \frac{\operatorname{erf}(6) \sqrt{\pi}}{2} \quad (16)$$

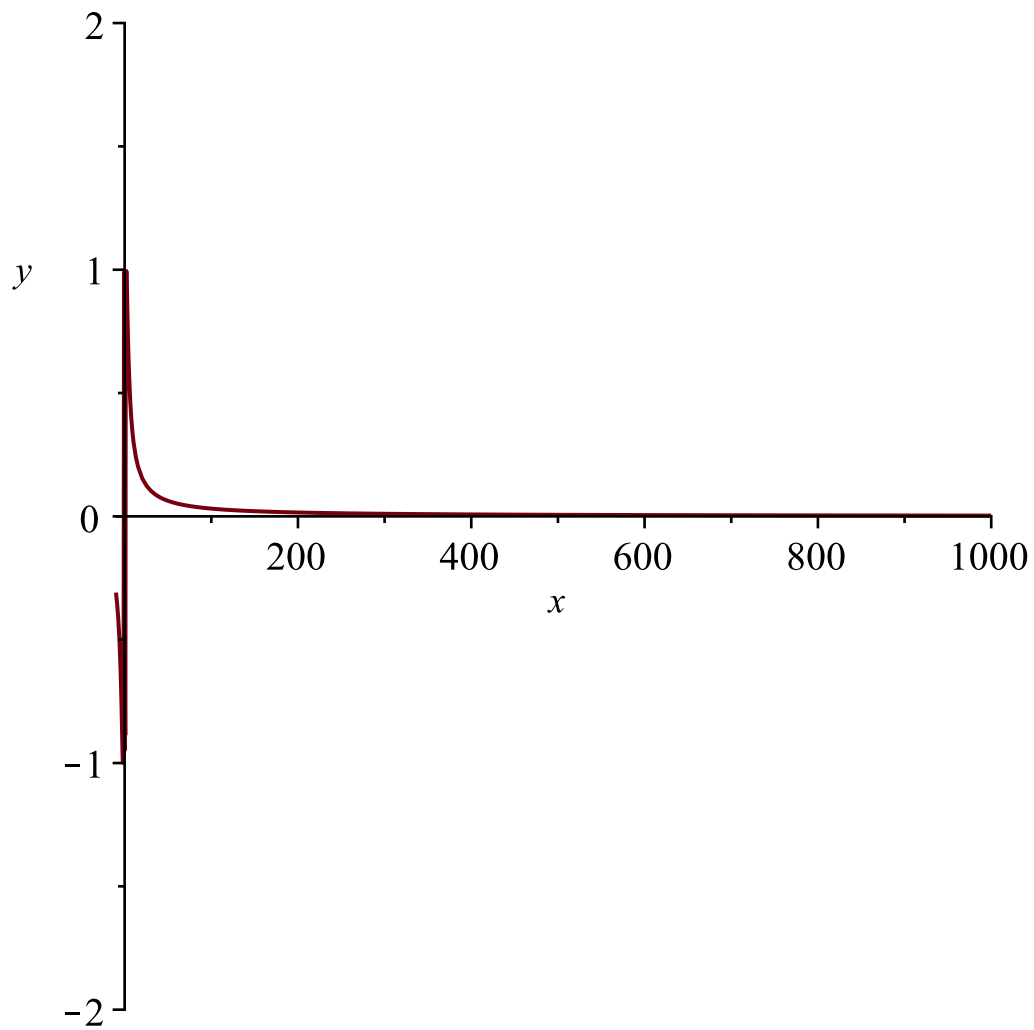
evalf(%)

$$1.768308316 \quad (17)$$

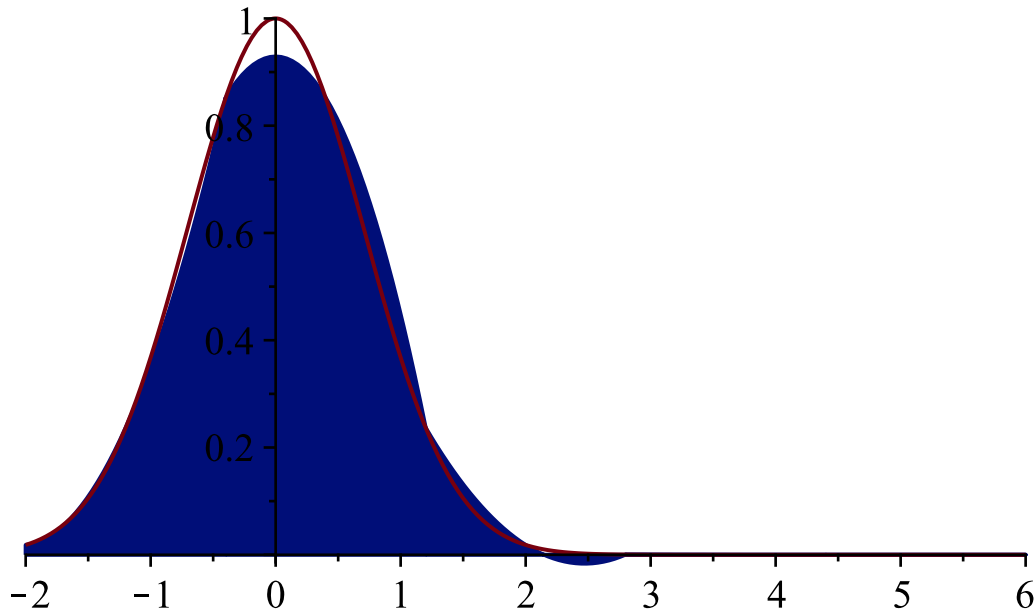
`with(Student[CalculusI]) :`

`with(plots) :`

`plot(sin(Pi/x), x=-10..1000, y=-2..2)`



ApproximateInt($\exp(-x^2)$, -2 .. 6, 'partition' = 10, 'method' = simpson, 'partitiontype' = normal, 'output' = 'plot', 'boxoptions' = ['filled' = ['transparency' = .5]]);



An approximation of $\int_{-2}^6 f(x) dx$ using Simpson's rule, where

$f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 1.767147796. Number of subintervals used: 5.

?ApproximateInt
f(x)

$$e^{-x^2} \quad (18)$$

fp := D(f)

$$fp := x \mapsto -2x e^{-x^2} \quad (19)$$

fp2 := D(fp)

$$fp2 := x \mapsto -2e^{-x^2} + 4x^2 e^{-x^2} \quad (20)$$

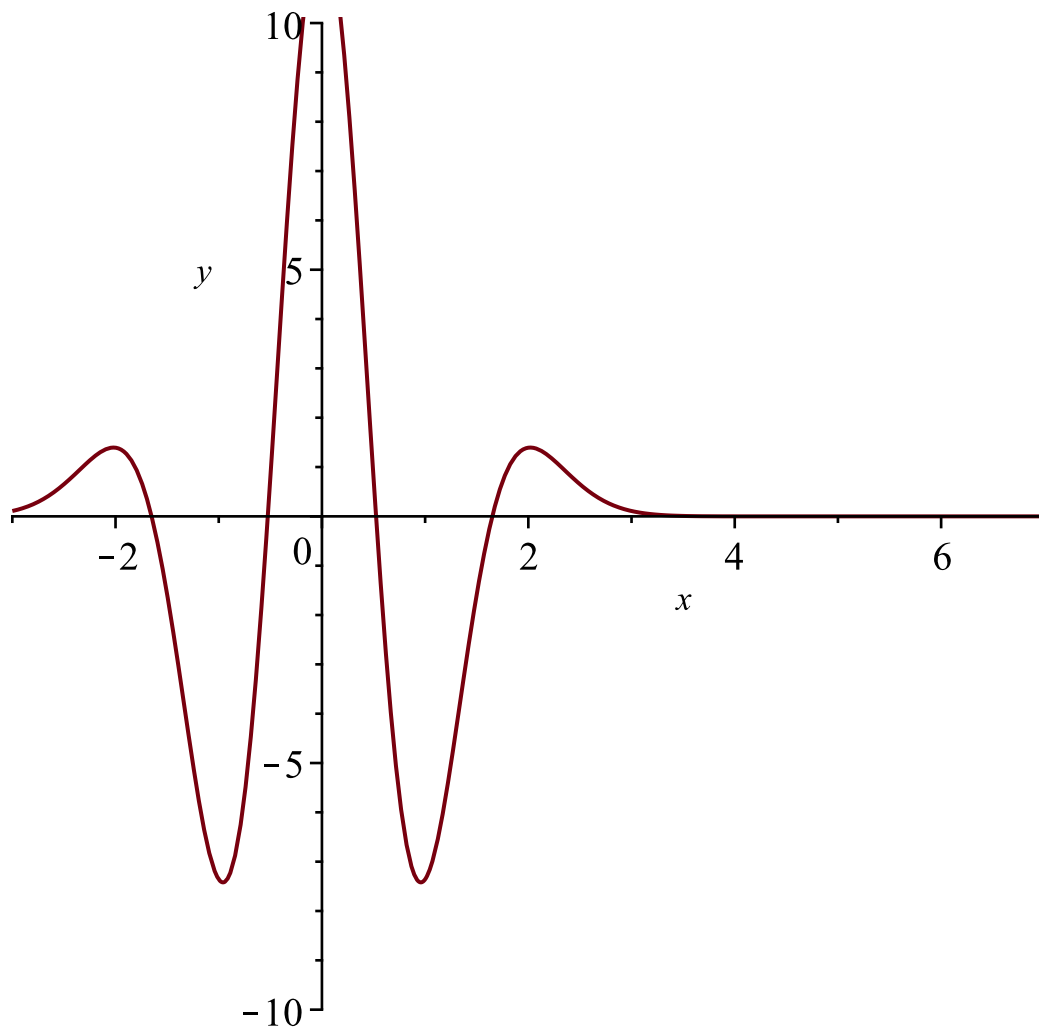
fp3 := D(fp2)

$$fp3 := x \mapsto 12x e^{-x^2} - 8x^3 e^{-x^2} \quad (21)$$

fp4 := D(fp3)

$$fp4 := x \mapsto 12e^{-x^2} - 48x^2 e^{-x^2} + 16x^4 e^{-x^2} \quad (22)$$

plot(fp4(x), x=-3..7, y=-10..10)



$$fp5 := D(fp4)$$

$$fp5 := x \mapsto -120 x e^{-x^2} + 160 x^3 e^{-x^2} - 32 x^5 e^{-x^2} \quad (23)$$

$$solve(fp5(x) = 0)$$

$$0, \frac{\sqrt{10 - 2\sqrt{10}}}{2}, -\frac{\sqrt{10 - 2\sqrt{10}}}{2}, \frac{\sqrt{10 + 2\sqrt{10}}}{2}, -\frac{\sqrt{10 + 2\sqrt{10}}}{2} \quad (24)$$

$$evalf(\%)$$

$$0., 0.9585724645, -0.9585724645, 2.020182870, -2.020182870 \quad (25)$$

$$fp4(-0.9585724645)$$

$$-7.419481177 \quad (26)$$

$$fp4(2.020182870)$$

$$1.394907051 \quad (27)$$

$$fp4(-2.020182870)$$

$$1.394907051 \quad (28)$$

$$fp4(-2)$$

$$76 e^{-4} \quad (29)$$

$$evalf(\%)$$

$$1.391988556 \quad (30)$$

$$fp4(6) \qquad \qquad \qquad 19020 e^{-36} \qquad \qquad \qquad (31)$$

$$evalf(\%) \qquad \qquad \qquad 4.411732423 \cdot 10^{-12} \qquad \qquad \qquad (32)$$

$$fp4(0) \qquad \qquad \qquad 12 \qquad \qquad \qquad (33)$$

$$evalf\left(\frac{8}{2930} \cdot \sum_{k=1}^{2930} f\left(-2 + \frac{k \cdot 8}{2930}\right)\right) \qquad \qquad \qquad 1.768283257 \qquad \qquad \qquad (34)$$

▼ **Dylan's 5-term Test 2 #3.**

$$\left[evalf\left(\frac{8}{5} \cdot \sum_{k=1}^5 f\left(-2 + \frac{k \cdot 8}{5}\right)\right) \right. \qquad \qquad \qquad 1.743144352 \qquad \qquad \qquad (1.1)$$

▼ **Kennan's 1000-term Test 2 #3.**

$$\left[evalf\left(\frac{8}{1000} \cdot \sum_{k=1}^{1000} f\left(-2 + \frac{k \cdot 8}{1000}\right)\right) \right.$$

▼ **Keenan's n=10, right-Riemann sum for Test 2 #1.**

$$g := x \rightarrow \text{sqrt}(1 - \sin(x)) \qquad \qquad \qquad g := x \mapsto \sqrt{1 - \sin(x)} \qquad \qquad \qquad (2.1)$$

$$a := \frac{2 \cdot \text{Pi}}{3} \qquad \qquad \qquad a := \frac{2 \pi}{3} \qquad \qquad \qquad (2.2)$$

$$b := \frac{4 \cdot \text{Pi}}{3} \qquad \qquad \qquad b := \frac{4 \pi}{3} \qquad \qquad \qquad (2.3)$$

$$n := 10 \qquad \qquad \qquad n := 10 \qquad \qquad \qquad (2.4)$$

$$\Delta x := \frac{(b - a)}{n} \qquad \qquad \qquad \Delta x := \frac{\pi}{15} \qquad \qquad \qquad (2.5)$$

$$evalf\left(\Delta x \cdot \sum_{k=1}^{10} g(a + k \cdot \Delta x)\right) \qquad \qquad \qquad 2.102891716 \qquad \qquad \qquad (2.6)$$

$$\int_{\frac{2 \cdot \text{Pi}}{3}}^{\frac{4 \cdot \text{Pi}}{3}} g(x) \, dx$$

2 (2.7)

evalf(%)

$$0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{11 \pi}{15} \right)} + 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{4 \pi}{5} \right)}$$

$$+ 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{13 \pi}{15} \right)} + 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{14 \pi}{15} \right)}$$

$$+ 0.2094395103 \sqrt{1. - 1. \sin(x) (\pi)} + 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{16 \pi}{15} \right)}$$

$$+ 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{17 \pi}{15} \right)} + 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{6 \pi}{5} \right)}$$

$$+ 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{19 \pi}{15} \right)} + 0.2094395103 \sqrt{1. - 1. \sin(x) \left(\frac{4 \pi}{3} \right)}$$

(35)

$$\int_{-2}^6 f(x) \, dx$$

$$\frac{\text{erf}(2) \sqrt{\pi}}{2} + \frac{\text{erf}(6) \sqrt{\pi}}{2}$$

(36)

evalf(%)

1.768308316 (37)