Section 11.1 - 11.7 Test is up.

I'll post another take for those who want one.

§11.8

$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$ Root Test

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{x^n}\right| = \sqrt[3]{\frac{n}{n+1}}\ |x| \xrightarrow{n\to\infty} |x|$

Want $\boxed{|x| < 1} \Rightarrow \underbrace{-1 < x < 1}_{\text{OK}}$ is fine.

$|x| < 1$

$x < 1$ and $x > -1$

GOOD

$|x| > 1$

$-1 > x > 1$ $\underline{\text{BAD}}$

To handle these, do

$x > 1$ OR $x < -1$

Says Radius of convergence is $r = 1$.

Check Endpoints

Centered at $x = 0$, $r = 1$,

Endpoints are ± 1

$x = 1$: $\sum_{n=1}^{\infty} \frac{(-1)(1)^n}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ $\quad a_n = \frac{-1}{\sqrt[3]{n}} \xrightarrow{n\to\infty} 0$ ✓

alternating $\Rightarrow$ converges.

$x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} \not\rightarrow$

So interval of convergence is

$I = (-1, 1]$

$$\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \sqrt{1-\sin x}\;dx$$

$$\sin x = \sin\left(2\left(\tfrac{x}{2}\right)\right)$$
$$= 2\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)$$

$$\sqrt{1-\sin x} = \sqrt{\sin^2\left(\tfrac{x}{2}\right) + \cos^2\left(\tfrac{x}{2}\right) - 2\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)}\;dx$$

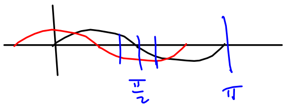
$$\sqrt{\sin^2\left(\tfrac{x}{2}\right) - 2\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right) + \cos^2\left(\tfrac{x}{2}\right)}$$

$$= \sqrt{\left(\sin\tfrac{x}{2} - \cos\tfrac{x}{2}\right)^2}$$

$$= \left|\sin\left(\tfrac{x}{2}\right) - \cos\left(\tfrac{x}{2}\right)\right| \quad \text{on } \left[\tfrac{2\pi}{3}, \tfrac{4\pi}{3}\right]$$

$$\tfrac{x}{2} \in \left[\tfrac{\pi}{3}, \tfrac{2\pi}{3}\right]$$

$$\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(\sin\left(\tfrac{x}{2}\right) - \cos\left(\tfrac{x}{2}\right)\right)dx$$



Repeating Decimals

$.21212121\ldots$

2 digits repeat

$x = .212121\ldots$

$100x = 21.212121\ldots$

$99x = 21$

$x = \frac{21}{99}$

$$\begin{array}{r} 358.358358358\ldots = 1000x \\ .358358358\ldots = x \\ \hline 358 = 999x \end{array}$$

$x = \frac{358}{999}$